Supplemental Problems for PHY 490

Do the problems in the list when assigned.

1. A runner carries a 20.0 m pole at a constant velocity in the direction of its length. It appears to be 10.0 m long to an observer at rest with respect to a nearby barn which is 10.0 m long. The runner enters the barn and just as the front end of the pole reaches the closed back end, the front door closes and an instant later the rear door opens as the runner continues through the barn.
    a. How long is the pole and how wide is the barn according to the runner?
    b. In part a you should find that the pole is too long to fit in the barn according to the runner. Quantitatively, and using spacetime diagrams, explain the apparent paradox.
    c. What does the observer see?
2. Consider the four-vectors $a^\alpha = (-2,0,0,1)$ and $b^\alpha = (5,0,3,4)$.
    a. Determine if these are timelike, spacelike, or null.
    b. Compute $\mathbf{a}\cdot\mathbf{b}$.
3. A beam of light enters a room on the Earth horizontally. How wide does the room need to be for the Earth's gravity to bend the beam by 1.0 m?
4. The metric on a 2D sphere is given by $dS^2 = R^2(d\theta^2 + \sin^2\theta\, d\phi^2)$. Calculate the Christoffel symbols from the metric.
5. Consider the parametrization of the cone, $x^2 + y^2 - k^2z^2 = 0$, given by $x = kh\cos\theta,\ y = kh\sin\theta,\ z = h$.
    a. Find the 2x2 metric for this surface.
    b. Write down the Lagrangian for this metric.
    c. Find the geodesic equations.
    d. Identify the Christoffel symbols.
6. The line element of a flat spacetime in a frame rotating with angular velocity Ω about the *z*-axis is given by $ds^2 = -\left[1-\Omega^2(x^2+y^2)\right]dt^2 + 2\Omega(y\,dx - x\,dy)dt + dx^2 + dy^2 + dz^2$.
    a. Find the geodesic equations.
    b. Identify the nonzero Christoffel symbols.