General Relativity and Cosmology – Part II

I. People
   a. William and Caroline Herschel
   b. Friedrich Bessel
   c. Immanuel Kant
   d. Vesto Slipher
   e. Henrietta Leavitt
   f. Vera Rubin
   g. Kent Ford
   h. Edwin Hubble
   i. Fritz Zwicky
   j. Sir Arthur Eddington

II. Objects
   a. Cepheids
   b. Coma Cluster
   c. Virgo Cluster
   d. Andromeda, M31
   e. Supernovae
   f. White-brown-red dwarfs, neutron stars, black holes,
   g. MACHOs, WIMPs
   h. Dark halo
   i. Clusters

III. Terms
   a. Event horizon
   b. Accretion disk
   c. Wormhole
   d. Scri-plus, Scri-minus
   e. Copernican Principle
   f. Hubble time, Hubble length
   g. Horizon
   h. Planck scales
   i. Chandrasekhar limit
   j. Baryonic
   k. Deceleration parameter
   l. Standard candle
   m. Lookback time
   n. Bolometer
   o. Luminosity, flux
   p. Big Crunch, Big Freeze, Big Bounce
   q. Loitering universe
   r. Benchmark Model
   s. BBN

IV. Schwarzschild Spacetime and Black Holes
   a. Metric

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(c\,dt)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

   b. Schwarzschild radius
   c. Embedding Diagrams, $ds^2 = \left[1 + \left(\frac{dz}{dr}\right)^2\right] dr^2 + r^2 d\phi^2$
      i. Black hole diagrams – Eddington-Finkelstein, Kruskal, Penrose

ii. Locate singularities, infinities, draw light cones, null geodesics.

d. GR Tests
   i. Gravitational lensing
   ii. Precession of Mercury's Perihelion
   iii. Orbit Equation, Effective Potential

$$E = \frac{1}{2} m\dot{r}^2 + \left(1 - \frac{r^*}{r}\right)\frac{\ell^2}{2mr^2} - \frac{GmM}{r}$$

   iv. Wormholes – characteristic features

V. Cosmology

a. Hubble Law: $v = H_0 r,\ z \approx \frac{v}{c}$, Hubble parameter: $H(t) = \frac{\dot{a}(t)}{a(t)}$

b. Hubble time: H_0^{-1}, Hubble length: $\frac{c}{H_0}$

c. Proper distance $d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$, $d_p(t_e) = \frac{d_p(t_0)}{1+z}$

d. Horizon distance: $d_p(t) = c\int_0^{t_0} \frac{dt}{a(t)}$,

e. Red shift parameter: $1 + z = \frac{a(t_0)}{a(t_e)},\ a(t_0) = 1.$

f. FLRW metric: $ds^2 = dr^2 + S_k(r)^2 d\Omega^2,\ ds^2 = \frac{dx^2}{1-\kappa x^2} + x^2 d\Omega^2$

g. Three types of spaces – flat, sphere, pseudosphere/hyperbolic

h. Friedmann Equation
   i. $H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R_0^2 a^2}$
   ii. $\frac{\kappa}{R_0^2} = H_0^2\left(1 - \Omega_0\right),\ H_0^2 = \frac{8\pi G}{3}\rho_0 - \frac{\kappa c^2}{R_0^2}$
   iii. $\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$,

i. Density parameters
   i. $\Omega = \frac{\rho}{\rho_c},\ \rho_c = \frac{3H^2}{8\pi G}, \rho_w = \rho_{w,0} a^{-3(1+w)} \Rightarrow \rho_m, \rho_r$
   ii. $\Omega_{CMB,0} = 5.0\times 10^{-5},\ \Omega_{\nu,0} = 0.681\Omega_{CMB}, \Omega_{r,0} = 8.4\times 10^{-5}$
   iii. $\Omega_{m,0} = 0.3, \Omega_{\Lambda,0} = 0.7$ (Benchmark)

j. Models
   i. Curvature Only (What happens? $a = ?, t_0 = ?, t_e = ?$)
   ii. Spatially Flat Universes
      1. $a(t) = \left(\frac{t}{t_e}\right)^{2/(3+3w)}$, $t_0 = \frac{2}{3(1+w)} H_0^{-1},\ \rho(t) = \frac{3}{8\pi G} H_0^2 \left(\frac{t}{t_0}\right)^{-2}$
         a. Lambda Only: $\dot{a} = H_0 a \Rightarrow a(t) = e^{H_0(t-t_0)}$

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iii. Multiple-Component Universes

1. $$H_0 t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2 + 1 - \Omega_0}}$$

2. Determining Epochs, $a_{rm}, a_{m\Lambda}$, t_{crunch}, etc.

VI. Luminosity

a. Flux $f = \frac{L}{4\pi S_\kappa^2(r)(1+z)^2}$

b. Luminosity distance $d_L = S_\kappa^2(r)(1+z), d_L = d(t_0)(1+z), [\kappa = 0]$

c. Standard Candles – Cepheids, Supernovae Type Ia - $L = 4\times10^9 L_{sun}$

d. Apparent magnitude $m = -2.5\log_{10}\frac{f}{f_0}$

e. Absolute magnitude $M = -2.5\log_{10}\frac{L}{L_0} = m - 5\log_{10}\frac{d_L}{10\,pc}$

VII. Big Bang Cosmology Timeline

a. Epochs

b. Baryon to photon ratio $\eta = 5\times10^{-5}$

c. (Radiative) Recombination epoch ($z = 1370, T = 3740K, t = 240,000yr$)

$p + e^- \leftrightarrow H + \gamma$

d. Photon decoupling ($z = 1100, T = 3000K, t = 350,000yr$)

Thomson scattering $\lambda + e^- \leftrightarrow \gamma + e^-$

e. Last scattering surface

f. Reionization

VIII. Big Bang Nucleosynthesis

a. Binding energies $p + e^- \leftrightarrow H + 13.6\text{eV}$ $p + n \leftrightarrow D + 2.22\text{MeV}$

b. $T_{nuc} = 6\times10^8 K, t_{nuc} = 300s$

c. He fraction, Y – primordial 24%, max 1/3

d. Beta decay $n \rightarrow p + e^- + \bar{\nu}_e$ ($\tau_n = 890s$)

e. Baryon-antibaryon asymmetry

IX. Inflation?

a. What is inflation?

b. Key problems – Flatness, Horizon, Monopole

c. Particle Physics Standard Model – Glashow, Weinberg, Salam, Gell-Mann, Higgs Quarks, Baryons, Mesons, Bosons (photons, gluons, W, Z), Leptons

d. Phase transitions (loss of symmetry) and unification of forces: electroweak, GUT and TOE, QED, QCD

e. e-foldings,

X. Dark Matter

a. Observational tests

b. Orbital speeds in spiral galaxies $v = \sqrt{\frac{GM}{R}}$