

General Relativity and Cosmology – Part II

- I. People
 - a. William and Caroline Herschel
 - b. Friedrich Bessel
 - c. Immanuel Kant
 - d. Vesto Slipher
 - e. Henrietta Leavitt
 - f. Vera Rubin
 - g. Kent Ford
 - h. Edwin Hubble
 - i. Fritz Zwicky
 - j. Sir Arthur Eddington
- II. Objects
 - a. Cepheids
 - b. Coma Cluster
 - c. Virgo Cluster
 - d. Andromeda, M31
 - e. Supernovae
 - f. White-brown-red dwarfs, neutron stars, black holes,
 - g. MACHOs, WIMPs
 - h. Dark halo
 - i. Clusters

- III. Terms
 - a. Event horizon
 - b. Accretion disk
 - c. Wormhole
 - d. Scri-plus, Scri-minus
 - e. Copernican Principle
 - f. Hubble time, Hubble length
 - g. Horizon
 - h. Planck scales
 - i. Chandrasekhar limit
 - j. Baryonic
 - k. Deceleration parameter
 - l. Standard candle
 - m. Lookback time
 - n. Bolometer
 - o. Luminosity, flux
 - p. Big Crunch, Big Freeze, Big Bounce
 - q. Loitering universe
 - r. Benchmark Model
 - s. BBN

- IV. Schwarzschild Spacetime and Black Holes
 - a. Metric

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) (c dt)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

- b. Schwarzschild radius

- c. Embedding Diagrams, $ds^2 = \left[1 + \left(\frac{dz}{dr}\right)^2\right] dr^2 + r^2 d\phi^2$

- i. Black hole diagrams – Eddington-Finkelstein, Kruskal, Penrose

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- ii. Locate singularities, infinities, draw light cones, null geodesics.
- d. GR Tests
 - i. Gravitational lensing
 - ii. Precession of Mercury's Perihelion
 - iii. Orbit Equation, Effective Potential

$$E = \frac{1}{2} m \dot{r}^2 + \left(1 - \frac{r^*}{r}\right) \frac{\ell^2}{2mr^2} - \frac{GmM}{r}$$

- iv. Wormholes – characteristic features

V. Cosmology

- a. Hubble Law: $v = H_0 r$, $z \approx \frac{v}{c}$, Hubble parameter: $H(t) = \frac{\dot{a}(t)}{a(t)}$
- b. Hubble time: H_0^{-1} , Hubble length: $\frac{c}{H_0}$
- c. Proper distance $d_p(t_0) = c \int_{t_e}^{t_0} \frac{dt}{a(t)}$, $d_p(t_e) = \frac{d_p(t_0)}{1+z}$
- d. Horizon distance: $d_p(t) = c \int_0^{t_0} \frac{dt}{a(t)}$,
- e. Red shift parameter: $1+z = \frac{a(t_0)}{a(t_e)}$, $a(t_0) = 1$.
- f. FLRW metric: $ds^2 = dr^2 + S_k(r)^2 d\Omega^2$, $ds^2 = \frac{dx^2}{1-\kappa x^2} + x^2 d\Omega^2$
- g. Three types of spaces – flat, sphere, pseudosphere/hyperbolic
- h. Friedmann Equation

$$i. H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{\kappa c^2}{R_0^2 a^2}$$

$$ii. \frac{\kappa}{R_0^2} = H_0^2 (1 - \Omega_0), \quad H_0^2 = \frac{8\pi G}{3} \rho_0 - \frac{\kappa c^2}{R_0^2}$$

$$iii. \frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1 - \Omega_0}{a^2},$$

i. Density parameters

$$i. \Omega = \frac{\rho}{\rho_c}, \quad \rho_c = \frac{3H^2}{8\pi G}, \quad \rho_w = \rho_{w,0} a^{-3(1+w)} \Rightarrow \rho_m, \rho_r$$

$$ii. \Omega_{CMB,0} = 5.0 \times 10^{-5}, \quad \Omega_{v,0} = 0.681 \Omega_{CMB}, \quad \Omega_{r,0} = 8.4 \times 10^{-5}$$

$$iii. \Omega_{m,0} = 0.3, \Omega_{\Lambda,0} = 0.7 \text{ (Benchmark)}$$

j. Models

- i. Curvature Only (What happens? $a = ?$, $t_0 = ?$, $t_e = ?$)

ii. Spatially Flat Universes

$$1. a(t) = \left(\frac{t}{t_e}\right)^{2/(3+3w)}, \quad t_0 = \frac{2}{3(1+w)} H_0^{-1}, \quad \rho(t) = \frac{3}{8\pi G} H_0^2 \left(\frac{t}{t_0}\right)^{-2}$$

$$a. \text{ Lambda Only: } \dot{a} = H_0 a \Rightarrow a(t) = e^{H_0(t-t_0)}$$

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iii. Multiple-Component Universes

$$1. \quad H_0 t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2 + 1 - \Omega_0}}$$

2. Determining Epochs, $a_{rm}, a_{m\Lambda}, t_{crunch}$, etc.

VI. Luminosity

a. Flux $f = \frac{L}{4\pi S_\kappa^2(r)(1+z)^2}$

b. Luminosity distance $d_L = S_\kappa^2(r)(1+z), d_L = d(t_0)(1+z), [\kappa = 0]$

c. Standard Candles – Cepheids, Supernovae Type Ia - $L = 4 \times 10^9 L_{sun}$

d. Apparent magnitude $m = -2.5 \log_{10} \frac{f}{f_0}$

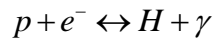
e. Absolute magnitude $M = -2.5 \log_{10} \frac{L}{L_0} = m - 5 \log_{10} \frac{d_L}{10 pc}$

VII. Big Bang Cosmology Timeline

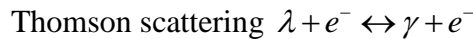
a. Epochs

b. Baryon to photon ratio $\eta = 5 \times 10^{-5}$

c. (Radiative) Recombination epoch ($z = 1370, T = 3740K, t = 240,000yr$)



d. Photon decoupling ($z = 1100, T = 3000K, t = 350,000yr$)



e. Last scattering surface

f. Reionization

VIII. Big Bang Nucleosynthesis

a. Binding energies $p + e^- \leftrightarrow H + 13.6eV$ $p + n \leftrightarrow D + 2.22MeV$

b. $T_{nuc} = 6 \times 10^8 K, t_{nuc} = 300s$

c. He fraction, Y – primordial 24%, max 1/3

d. Beta decay $n \rightarrow p + e^- + \bar{\nu}_e$ ($\tau_n = 890s$)

e. Baryon-antibaryon asymmetry

IX. Inflation?

a. What is inflation?

b. Key problems – Flatness, Horizon, Monopole

c. Particle Physics Standard Model – Glashow, Weinberg, Salam, Gell-Mann, Higgs
Quarks, Baryons, Mesons, Bosons (photons, gluons, W, Z), Leptons

d. Phase transitions (loss of symmetry) and unification of forces: electroweak, GUT
and TOE, QED, QCD

e. e-foldings,

X. Dark Matter

a. Observational tests

b. Orbital speeds in spiral galaxies $v = \sqrt{\frac{GM}{R}}$