PHY 490 Final Exam **Name _____**

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- All short answers and essays should be responded to with full sentences conveying thoughtful responses.
- If you need more space, you may use the **back of a page** and write *On back of page #* in the problem space or the **extra page**. **No other paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient rationale for full credit.

Use proper significant digits and units.

Pay attention to the point distribution. Not all problems have the same weight. Pace yourself!

Page	Pts	Score
1	16	
2	18	
3	16	
4	13	
5	11	
6	19	
7	15	
8	12	
Total	120	

Useful information - $L_0 = 78.7 L_{sun}$, $f_0 = 2.53\times10^{-8}\,\mathrm{Wm^{-2}}$, $M_{sun} = 1.989\times10^{30}\,\mathrm{kg}$,
$M_{earth} = 5.98\times10^{24}\,\mathrm{kg}$, $1\,Mpc = 3.086\times10^{22}\,\mathrm{m}$, $1\,AU = 1.496\times10^{8}\ \mathrm{km}$
$G = 6.673\times10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$, $h = 6.626\times10^{-34}\,\mathrm{Js}$, $\varepsilon_{c,0} = 8.3\times10^{-10}\,\mathrm{Jm^{-3}}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R_0^2 a^2}, \quad g_{\alpha\delta}\Gamma^{\delta}_{\beta\gamma} = \frac{1}{2}\left(\frac{\partial g_{\alpha\beta}}{\partial x^{\gamma}} + \frac{\partial g_{\alpha\gamma}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}}\right)$$

PHY 490 Final Exam **Name _____**

1. (5 pts) Name the key discovery of the following and their colleagues:

    a. Zwicky

    b. Herschel

    c. Penzias

    d. Pound

    e. Eddington

2. (5 pts) Define the following:
    a. Homogeneous Spacetime

    b. Static Spacetime

    c. Test particle

    d. CMBR

    e. Critical Density

3. (3 pts) Define the principle:
    a. Principle of Equivalence

    b. Principle of Relativity

    c. Cosmological Principle

4. (3 pts) Provide the following based on our current observations/theory:

    a. How old is the universe? _____

    b. Hubble's constant. _____

    c. What is the current CMB temperature? _____

5. (8 pts) Give exact expressions for the following:

   a. Minkowski spacetime in spherical coordinates.

   b. Schwarzschild metric.

   c. Einstein's Equation.

   d. General geodesic equation.

6. (3 pts) Which of the following are not a correct use of indices: _____
   a. $\Gamma^{\alpha}_{\alpha\gamma} a^{\gamma} = g_{\alpha\beta} a^{\alpha} b^{\beta}$.
   b. $g_{\alpha\beta} a^{\alpha} b^{\beta} = g_{\beta\gamma} a^{\beta} b^{\gamma}$.
   c. $\Gamma^{\beta}_{\alpha\beta} = \Gamma^{\beta}_{\beta\beta}$.

7. (4 pts) Let $a^{\alpha} = (2,-1,0,1)$ and $b^{\alpha} = (2,1,4,0)$.
   a. Are these four-vectors timelike, spacelike, or null? a: _____ b: _____

   b. Compute $\mathbf{a} \cdot \mathbf{b}$.

8. (3 pts) Prove that the Friedmann equation at the current time can be written as $\frac{kc^2}{R_0^2} = (\Omega_0 - 1) H_0^2$.

9. (6 pts) A measured red shift for a particular star is measured as 1.5.
   a. If the star emitted a wavelength of 450 nm, then what wavelength was observed?

   b. What was $a(t_e)$ for the universe at a time t_e when the light was emitted?

10. (3 pts) A signal of frequency 700 MHz is emitted from the ground floor of a building. An antenna receives it at the top of the tower (400 m). What is the fractional change in the frequency received?

11. (3 pts) Find the Schwarzschild radius of the Earth.

12. (4 pts) Let $(x, ct) = (120, 150)$ m in System S. In a system moving at $0.6c$ with respect to S, what are the measured coordinates (x', ct')?

13. (3 pts) Prove that the four-velocity of a particle satisfies $\mathbf{u}\cdot\mathbf{u} = -c^2$.

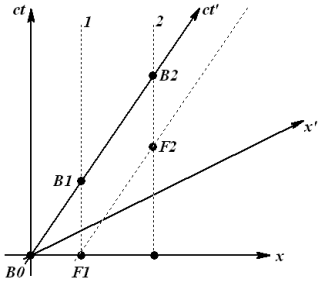
14. (3 pts) Let two events occur in Minkowski space at (3, -1, 2, 4) and (0, 4, -1, 1). Find Δs.

15. (4 pts) A star has luminosity $L = 400L_{sun}$.
    a. What is the absolute magnitude?

    b. If the apparent magnitude of the star in Problem 6 is 3.50, what is the luminosity distance?

16. (3 pts) If $ds^2 = dr^2 + r^2 d\phi^2 + dz^2 - 2drdz$, give the metric tensor.

17. (8 pts) A relativistic train of rest length 200 meters travels at 0.8*c* through a tunnel which has rest length 300 meters. In the figure below the world lines for the tunnel openings are drawn as line 1 and 2 and the world line of the front of the train is the third doted line. Let S_{tunnel} be the tunnel with coordinates (x,t) and let S_{train} be the train coordinates (x',t'). We set the origin as the event B0, the back of the train location just as the front end enters opening 1.



Determine the coordinates of the following for the given frame: Use units with $c=1$.

a. The coordinates (x, t) of F2 in S_{tunnel}

b. B1's coordinates in the *train* frame of reference

c. Find the coordinates of B1 in S_{tunnel}.

d. Use the Lorentz equations to find the coordinates of F2 in S_{train}.

18. (3 pts) Let $ds^2 = -2dudv + (1-u)^2 dx^2 + (1+u)^2 dy^2$. Find Γ^v_{xx}.

19. (3 pts) Consider a positively curved universe which contains only radiation. Find *a(t)*.

20. (8 pts) Let $ds^2 = -X^2 dT^2 + dX^2$.
    a. Write the Lagrangian.

    b. Find the geodesic equations for *X* and *T*.

    c. From the geodesic equations, determine Γ^T_{XT} and Γ^X_{TT}.

21. (8 pts) Answer the following:
    a. A vector which transforms from system *x* to system *y* via the transformation $V^{(y)}_{\alpha} = \frac{\partial y^{\beta}}{\partial x^{\alpha}} V^{(x)}_{\beta}$ is called _____.
    b. What tests confirm the theory of general relativity?

    c. What year is the centennial year for general relativity?

    d. What is MOND?

    e. What does TOE stand for?

    f. When did Big Bang Nucleosynthesis end?

22. (8 pts) GPS satellites are located at 26560 km from the center of the Earth, moving at 3.9 km/s. These orbits have a period of motion of 11 hours and 58 minutes. Their operation depends on time measurements, but you know that there are relativistic differences between the time recorded on the satellite and that on the Earth.
    a. Due to special relativity, would the clocks tick faster or slower on the satellite?

    b. Due to general relativity, would the clocks tick faster or slower on the satellite?

    c. What is the ratio of time interval measurements ($\Delta\tau_s / \Delta\tau_E$)
        i. Due to special relativity?

        ii. Due to general relativity?

23. (4 pts) Consider the worldline given by $x^\alpha = (\sinh\tau, \cosh\tau, 0, 0)$.
    a. Determine the four-velocity.

    b. Is this path time-like, space-like, or null? _____

24. (3 pts) What was the scale factor at the time of last scattering? ($z = 1100.$)

PHY 490 Final Exam **Name** _____

25. (6 pts) Describe these pictures as specifically as you can.

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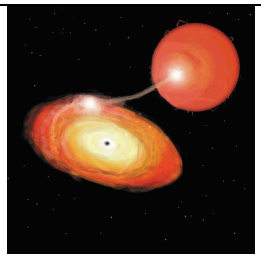

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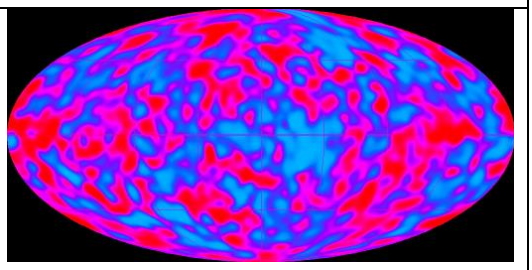

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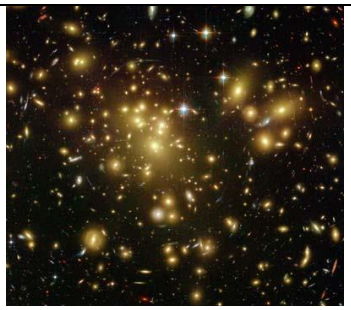

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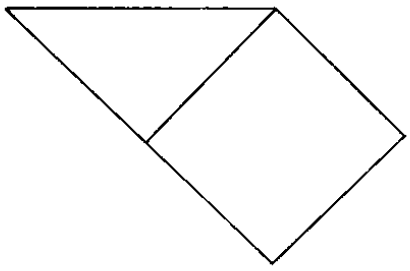
26. (6 pts) In the Penrose diagram for a Schwarschild black hole, indicate the following: The singularity, the event horizon, future null infinity, past null infinity, future timelike infinity, spacelike infinity.



Extra Page