PHY 490 Exam I

Name ____________________

1. (8 pts) Provide the following based on our current observations/theory:

   a. How old is the universe? 13.77×10^9 yr

   b. How many galaxies are there? 10^{11}

   c. Hubble's constant. 73.9 Km/s / Mpc

   d. What is the current CMB temperature? 2.725 K

2. (6 pts) M31 is 778 kpc from us.

   a. What is the common name of M31? Andromeda

   b. How fast should M31 be receding from us according to Hubble's Law?

   $V = H_0 r \approx 70 \frac{\text{Km/s}}{\text{Mpc}} (778 \text{ Kpc}) \sim 54 \text{ Km/s}$

   c. The redshift parameter for M31 is $z = -0.001001$. What does this tell you about the speed of M31 relative to the Earth?

   $z \simeq V/c$

   $v \sim 300 \text{ Km/s}$

   $z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} < 0 \Rightarrow \lambda_{obs} < \lambda_{em}$

   blueshifted

3. (2 pts) Describe the following:

   a. Homogeneous

   Define

   b. Isotropic

4. (6 pts) Give exact expressions for the following:

   a. The 2D spatial line element in polar coordinates.

   $ds^2 = dr^2 + r^2 d\theta^2$

   b. Minkowski spacetime in Cartesian coordinates.

   $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$

   c. Minkowski spacetime in spherical coordinates.

   $ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)$

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5. (3 pts) Which of the following are a correct use of indices: $\underline{\ b\ }$
   a. $g_{\alpha\beta}dx^{\alpha}dx^{\beta} = g_{\alpha\beta}dx^{\alpha}dx^{\gamma}$. X
   b. $g_{\alpha\beta}a^{\alpha}b^{\beta} = g_{\beta\gamma}a^{\beta}b^{\gamma}$.
   c. $\Gamma^{\beta}_{\alpha\beta} = \Gamma^{\beta}_{\beta\beta}$. X

6. (4 pts) Let $a^{\alpha} = (-2,0,0,1)$ and $b^{\alpha} = (5,0,3,4)$. HW!
   a. Are these four-vectors timelike, spacelike, or null? a: timelike b: null

   $a^a a^a = -(-2)^2 + 1^2 = -3$ $\quad$ $b^a b^a = -(5)^2 + 3^2 + 4^2 = 0$

   b. Compute $\mathbf{a} \cdot \mathbf{b}$.

   $a^a b^a = -(-2)(5) + 4 = \boxed{14}$

7. (3 pts) At what speed does a clock move if it runs one half the rate of a clock at rest?

   Then $\gamma = 2$

   $2 = \frac{1}{\sqrt{1-\beta^2}}$ $\quad$ $\beta = \sqrt{3}/2$ or $\boxed{v = \frac{\sqrt{3}}{2}c}$

   $1 - \beta^2 = \frac{1}{4}$

8. (3 pts) Two rockets of length L_0 approach each other from opposite directions at speed $c/2$ relative to an observer on Earth. How long (in terms of L_0) does a rocket appear to an observer on the other rocket?

   $c/2$ $\quad$ $-c/2$

   $u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{-c/2 - c/2}{1 + \frac{(c/2)^2}{c^2}} = -\frac{c}{5/4} = -\frac{4}{5}c \Rightarrow \beta = 4/5$

   E

   $\gamma = 5/3 \Rightarrow L = \frac{L_0}{\gamma} = \boxed{\frac{3}{5}L_0}$

9. (4 pts) A signal of frequency 900 MHz is emitted from the ground floor of the Sears (Willis) Tower in Chicago. An antenna receives it at the top of the tower (530 m). What is the fractional change in the frequency received? Is it blueshifted or redshifted when received? ______________

   530m

   $\frac{\Delta\omega}{\omega} = \frac{\omega_{rec} - \omega_{em}}{\omega_{em}} = -\frac{\Phi_{rec} - \Phi_{em}}{c^2} \Rightarrow \left|\frac{\Delta\omega}{\omega}\right| = \left|\frac{gh}{c^2}\right| \sim 5.8 \times 10^{-14}$

   $\Phi_{em} = g h_{em} < \Phi_{rec}$ $\quad$ redshifted

10. (3 pts) A triangle is drawn on a ball of radius 5.0 cm. The sum of the interior angles is $5\pi/2$. What fraction of the surface area does the triangle take up?

   $\alpha + \beta + \gamma = \pi + \frac{\sigma}{R^2}$

   Want $\frac{\sigma}{4\pi R^2} = \frac{1}{4\pi}(\alpha + \beta + \gamma - \pi) = \frac{1}{4\pi}\left(\frac{3\pi}{2}\right) = \boxed{\frac{3}{8}}$

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11. (4 pts) An astronaut orbits a 10 M_{sun} black hole at a radius of 35 km for 1 year (on her watch). She then returns to the parent spaceship, located very far from the black hole. How much time elapsed at the spaceship while orbiting the black hole?

$\Delta t = \left(1 + \frac{GM}{c^2 R}\right)\Delta t$ Clock slower near black hole

$\frac{GM}{c^2 R} = \frac{6.67\times10^{-11}\ 10\,(1.989\times10^{30})}{(3\times10^{8})^2\ 3500} = 4.2\times10^{-4}\ 10^{-11+31-16} = 4.2$

$\Rightarrow$ factor of $1+4.2 = 5.2 \Rightarrow \boxed{5.2\text{yrs}}$

12. (4 pts) Let $(x, ct) = (240, 300)$ m in System S. In a system moving at $0.8c$ with respect to S, what are the measured coordinates (x', ct')?

$x' = \gamma(x - vt) = \frac{5}{3}(240 - .8(300)) = 0$

$ct' = \gamma(ct - \frac{v}{c}x) = \frac{5}{3}(300 - .8(240)) = 180$

13. (4 pts) Consider the worldline given by $x^\alpha = (\sinh\tau, \cosh\tau, 0, 0)$. Determine the four-velocity. Is this path time-like, space-like, or null? time-like

$U^\alpha = \frac{dx^\alpha}{d\tau} = (\cosh\tau, \sinh\tau, 0, 0)$

$g_{\alpha\beta}U^\alpha U^\beta = -\cosh^2\tau + \sinh^2\tau = -1 < 0$

14. (3 pts) Consider the metric tensor given by $g_{\mu\nu} = \begin{pmatrix} 0 & f(r) & 0 & 0 \\ f(r) & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$ for a given set of coordinates (U, V, θ, ϕ), Write out the corresponding line element.

$ds^2 = 2f(r)\,dU\,dV + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$

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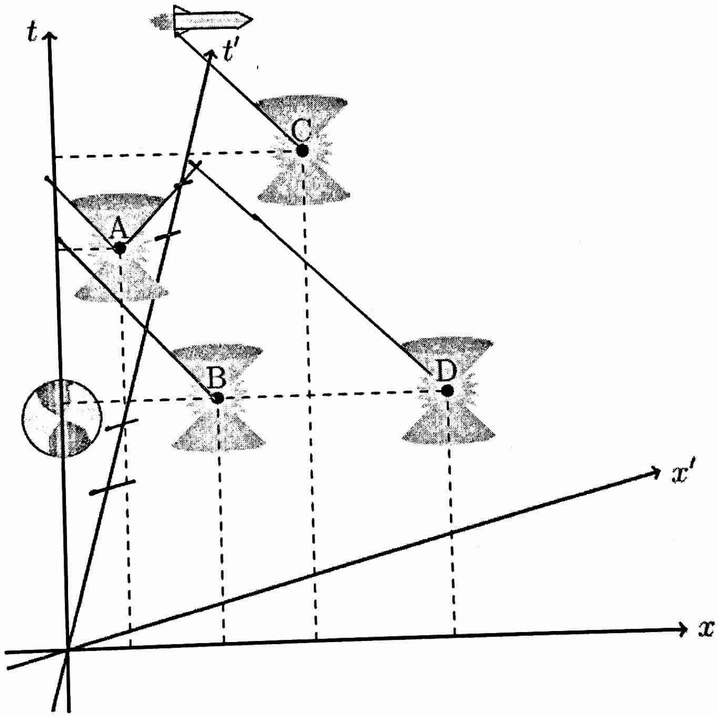
15. (3 pts) Prove that the four-velocity of a particle satisfies $\mathbf{u}\cdot\mathbf{u}=-1$. (if $c\equiv1$)

$$\vec{u} = \frac{dx^\mu}{d\tau} = \gamma\frac{dx^\mu}{dt} = \gamma(c, v^1, v^2, v^3)$$

$$|\vec{u}|^2 = \gamma^2(-c^2+v^2) = \frac{-c^2+v^2}{1-v^2/c^2} = -c^2$$

16. (10 pts) The spacetime diagram shows four supernovae events at A, B, C, and D. These supernovae are observed on the Earth (system S) and on a fast moving spaceship (system S'). Note that the worldlines of the Earth and spaceship start at $(x,t)=(x',t')=(0,0)$.



Answer the following questions:

- In which chronological order do the supernovae occur in the Earth frame of reference? B-D same, A, C
- In which chronological order do the supernovae occur in the spaceship frame of reference? D, B, A, C
- In which chronological order do astronomers on Earth see the supernovae?
  B A D C
- In which chronological order do observers on the spaceship see the supernovae?
  B A D C
- Is the chronological order in which supernovae A and B occur the same in all frames of reference? Explain. ___________