

Instructions:

- Place your name on all of the pages.
- Do all of your work in this booklet. Do not tear off any sheets.
- Show all of your steps in the problems for full credit.
- Be clear and neat in your work. Any illegible work, or scribbling in the margins, will not be graded.
- Put a box around your answers when appropriate..
- If you need more space, you may use the back of a page and write *On back of page #* in the problem space or the attached blank sheet. **No other scratch paper is allowed.**

Try to answer as many problems as possible. Provide as much information as possible. Show sufficient work or rationale for full credit. Remember that some problems may require less work than brute force methods.

If you are stuck, or running out of time, indicate as completely as possible, the methods and steps you would take to tackle the problem. Also, indicate any relevant information that you would use. Do not spend too much time on one problem. **Pace yourself.**

Pay attention to the point distribution. Not all problems have the same weight. Be careful to note **units** and use an appropriate number of **significant digits**.

Page	Pts	Score
1	17	
2	13	
3	14	
4	9	
5	17	
Total	70	

Constants: $L_0 = 78.7L_{sun}$, $f_0 = 2.53 \times 10^{-8} \text{ Wm}^{-2}$, $M_{sun} = 1.989 \times 10^{30} \text{ kg}$,

$M_{earth} = 5.98 \times 10^{24} \text{ kg}$, $1 \text{ Mpc} = 3.086 \times 10^{22} \text{ m}$, $1 \text{ AU} = 1.496 \times 10^8 \text{ km}$

$G = 6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, $h = 6.626 \times 10^{-34} \text{ Js}$, $\epsilon_{c,0} = 8.3 \times 10^{-10} \text{ Jm}^{-3}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho - \frac{\kappa c^2}{R_0^2 a^2}$$

Bonus: If the apparent magnitude of the star in Problem 6 is 2.50, what is the luminosity distance?

1. (4 pts) Give the Benchmark Model values:
 - a. $\Omega_{r,0} =$ _____
 - b. $\Omega_{m,0} =$ _____
 - c. $\Omega_{\Lambda,0} =$ _____

2. (9 pts) Answer the following:
 - a. _____ assigned the original magnitude range for stars.
 - b. The binding energy of deuterium is _____.
 - c. After studying the Coma cluster, _____ introduced the idea of dark matter.
 - d. The Cepheids were first classified by _____ as standard candles.
 - e. _____ was the first to measure red shifts of nebulae.
 - f. The _____ model was proposed by Fred Hoyle and others in the late 1940s in competition with the Big Bang Model.
 - g. A flat, matter dominated universe will end in a _____.
 - h. The _____ gives the maximum mass of a stable white dwarf star at $1.4 M_{\text{sun}}$.
 - i. _____ is a term used for future null infinity.

3. (2 pts) Give the Schwarzschild line element.

4. (2 pts) The equation of state takes the form $P = w\varepsilon = w\rho c^2$. What is w for
 - a. A photon gas? _____
 - b. Hydrogen gas? _____

5. (10 pts) You land on a planet that is two Schwarzschild radii from a black hole and has a mass comparable to that of the Earth. In honor of the recent death of an iconic star, you call the planet Vulcan.
- Assuming the black hole is very far from Earth, how much time passes on Earth for each hour that passes on Vulcan?
 - If Vulcan is one astronomical unit from the black hole, what is the mass of the black hole?
 - Assuming that Vulcan's orbit is circular, find an expression for the radius of the orbit. What is the value of Vulcan's orbital angular momentum?

Recall that the effective potential is given by $V_{eff} = \frac{\ell^2}{2mr^2} - \frac{r_s mc^2}{2r} - \frac{r_s \ell^2}{2mr^3}$,

where m is the planet mass.

6. (3 pts) A star has luminosity $L = 250L_{sun}$. What is the absolute magnitude?

7. (9 pts) Consider an empty, negatively curved universe.

a. Find $a(t)$ in terms of t and $t_0 = \frac{R_0}{c}$.

b. Find the proper distance $d_p(t_0)$.

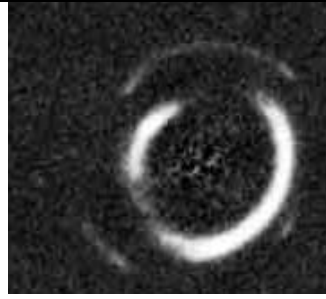
c. If the measured red shift for a particular star is given as 2.0, then determine $d_p(t_e)$ as a multiple of the Hubble distance.

8. (5 pts) Fill in the blanks in this timeline.

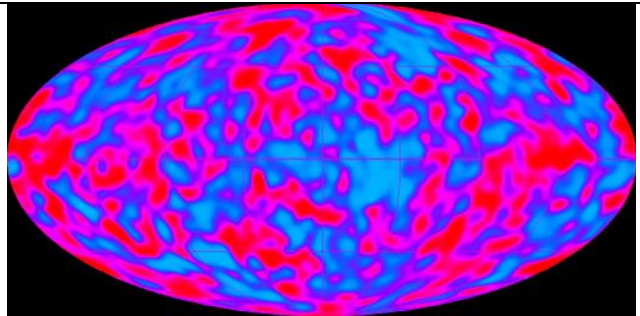
Time	a	Temp (K)	z	Event
13.5 Gy		2.7		Now
9.8Gy	0.75			Λ -Dominant
950 Myr			6	
300 Myr			14	Reionization
0.35 Myr			1100	
0.35 Myr		3000		Decoupling
0.24 Myr		3740	1370	
0.047 Myr	0.00028			Photoionization
		7.6×10^8		BBN

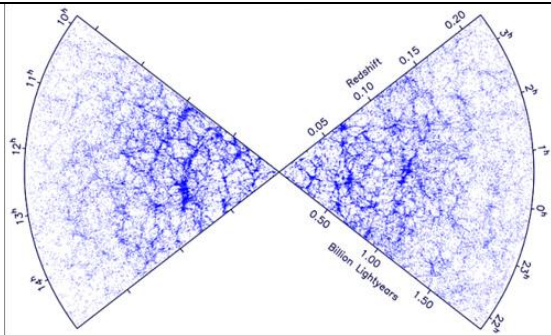
9. (6 pts) Describe these pictures as specifically as you can.









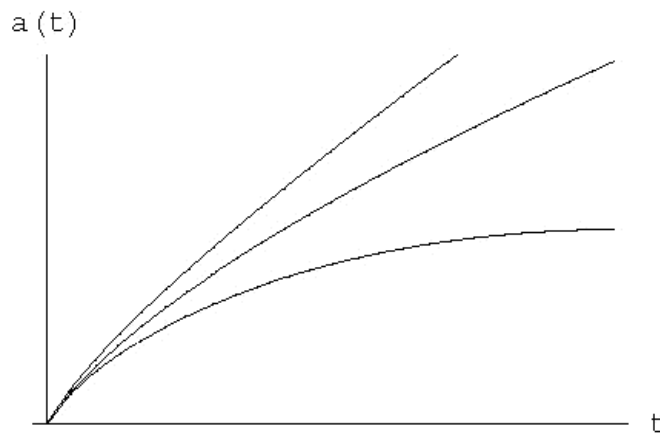




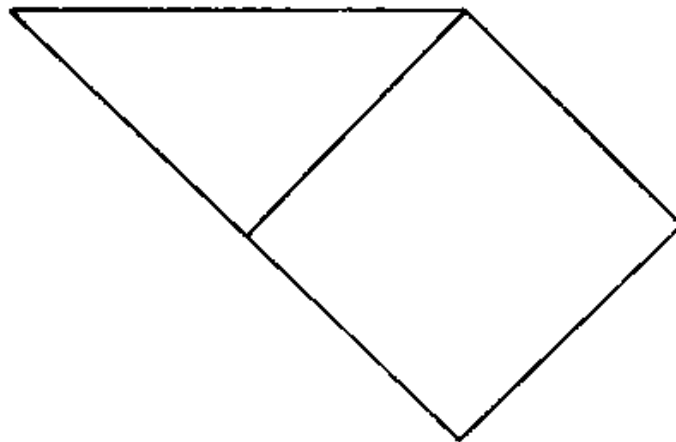
10. (3 pts) Rewrite the Friedmann equation at the current time in terms of H_0 and Ω_0 .

11. (3 pts) Consider the cosmic microwave background temperature. Using $\epsilon_\gamma = \alpha T^4$, $\alpha = 7.56 \times 10^{-16} \text{ Jm}^{-3} \text{ K}^{-4}$, find $\Omega_{\gamma,0}$.

12. (6 pts) On the plot below indicate the solutions corresponding to positive, zero, and negative curvature for a matter dominated FLRW spacetime. How does each universe end?



13. (8 pts) In the Penrose diagram for a Schwarzschild black hole, indicate the following: The singularity, the event horizon, future null infinity, past null infinity, future timelike infinity, spacelike infinity, and a path of constant r outside the event horizon.



Extra Space