PHY 490 Exam #1 **Name _____**

Column	Points	Score
1	14	
2	13	
3	14	
4	9	
Total	50	

Instructions:

1. Do all of your work in this booklet.
2. **Show all of your steps** in problems for full credit.
3. **Be clear and neat** in your work. Any illegible work, or scribbling in the margins, will not be graded.
4. Place your **answers in a box**.
5. If you need more space, you may **use the back of the page** and write **On back Page #** in the problem space.

1. Short Answers (4 pts)

a. What are Einstein's two postulates?

b. What is an inertial frame?

2. Formulae (10 pts) – Give the exact expressions for

a. The line element for the 2D surface of a sphere.

b. The gravitational potential due to a continuous mass distribution of density $\mu(\mathbf{x})$.

c. The action.

d. The metric for a flat 4D spacetime.

e. The explicit components of the four-momentum

3. Short Problems (13 Pts)

a. A triangle is drawn on a ball of radius 5.0 cm. The sum of the interior angles is $5\pi/2$. What fraction of the surface area does the triangle take up?

b. At what fraction of the speed of light must a particle move so that its kinetic energy is just equal to its rest energy

c. Prove that the four velocity is a time-like unit vector.

d. The Lagrangian for a particle attached to a spring in gravity is given by $L = \frac{1}{2}m\dot{z}^2 - mgz - \frac{1}{2}kz^2$, where g is the acceleration due to gravity and k is the spring constant. Find (but do not solve) the equation of motion of the particle.

e. An observer in frame S holds a 1.00 m stick at 45^o with respect to the positive x-axis. If S' is moving at $0.8c$ with respect to S, what is the length and angle of the meter stick measured by an observer at rest with respect to S' ?

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4. Leaving on a Fast Train (14 pts)

A relativistic train of rest length 240 meters travels at 0.6*c* through a tunnel which has rest length 360 meters. In the figure below the world lines for the tunnel openings are drawn as line 1 and 2 and the world line of the front of the train is the third doted line. Let S_{tunnel} be the tunnel with coordinates (x,t) and let S_{train} be the train coordinates (x',t'). We set the origin as the event B0, the back of the train location just as the front end enters opening 1.

a. Label the following events on the spacetime diagram:
F1: The front of the train enters door 1.
F2: The front of the train passes door 2.
B1: The back of the train enters the tunnel.
B2: The back of the train leaves the tunnel.

b. Determine the coordinates of the following for the given frame: Use units with $c = 1$.

i. The coordinates (x, t) of F2 in S_{tunnel}

ii. B1's coordinates in the *train* frame of reference

iii. Use the Lorentz equations to find the coordinates of B1 in S_{tunnel}. Is B1 before or after F2 in this frame? _____

iv. Use the Lorentz equations to find the coordinates of F2 in S_{train}.
Is B1 before or after F2 in this frame? _____

5. Winding to the End (9 pts)

Consider the parametrized world line $x^{\alpha} = (\sinh\tau, \cosh\tau, 0, 0)$.

a. Find the four-velocity.

b. Find the four-acceleration.

c. Determine the three-velocity.

d. What is the magnitude of the four-force? Simplify!

e. Is this path space-like or time-like? _____