Cooking with A Pinch of Physics and a Dash of Math

UNCW College Day, 2015

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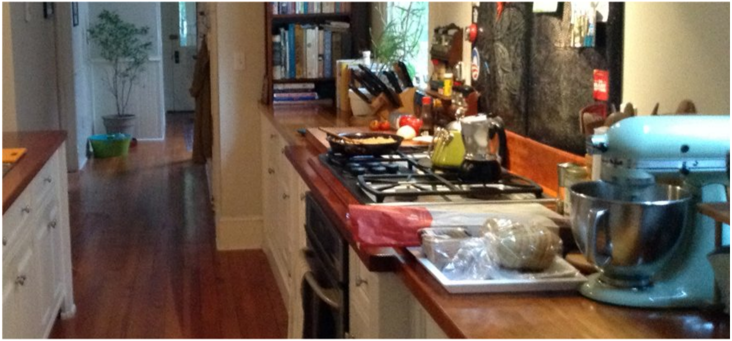
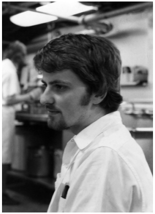
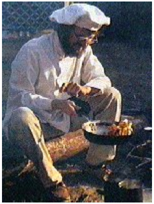
UNCW Mathematics & Statistics/Physics & Physical Oceanography

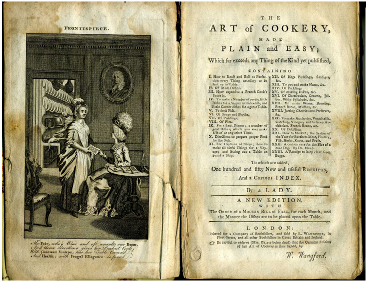


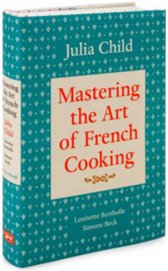
Outline





Magic, Art, or Science?



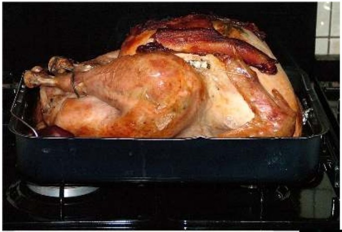
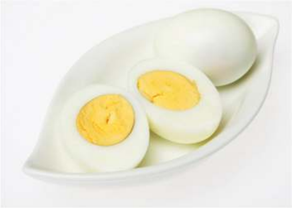


Cooking Methods

- Dry Heat
  - Baking
  - Steaming
  - Grilling
  - Roasting
- Moist Heat
  - Boiling
  - Stewing
  - Frying
    - Deep
    - Shallow
  - Barbecue
  - Basting

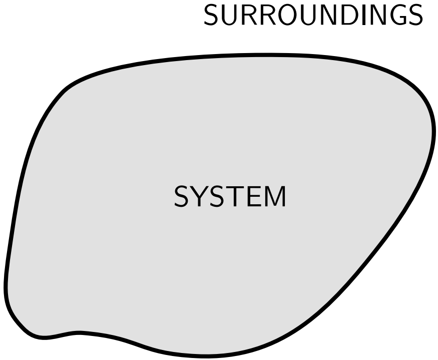
Cooking Times

- How long do I cook ...
- Molecular gastronomy - what happens at molecular level?
- Heating processes, or thermodynamics.

Laws of Thermodynamics

Developed in 1800's:



I. Conservation of Energy.

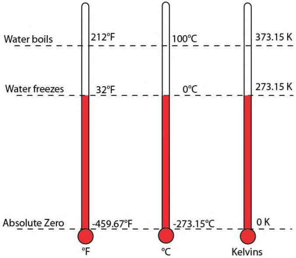
II. Cannot turn all heat energy to useful work.

III. Cannot get to absolute zero.

Temperature

Heat & temperature known, not understood - needed a reliable thermometer. (Conversion)

- Gabriel Daniel Fahrenheit (1686-1736), German instrument maker/physicist
- 1714 - Developed closed tube, mercury thermometer.
- 1717 - Went commercial.
- 1724 - Fahrenheit scale: melting of ice 32 F boiling water 212 F.



Heat energy

- What was heat? Two views:
  - Heat fluid in body, caloric.
  - Vibrations of atomic matter.
- Heat conduction in thin bar heated at one end - Jean Baptiste Biot (1774-1862)
- Heat conduction and heat loss (Newton's Law of Cooling)
  - The rate of change of the temperature is proportional to the temperature difference of the body and its surroundings.
  - 1800's - Stefan-Boltzmann Law of Radiation, radiant energy $\propto T^4$.

Heat Transfer

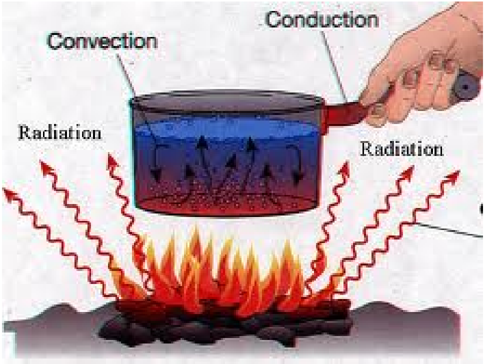
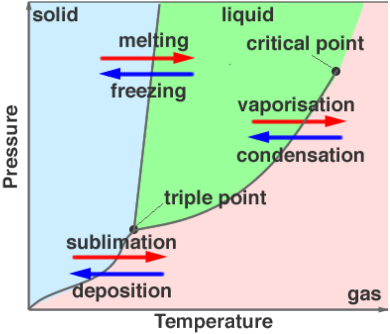


Figure: Conduction, Convection, Radiation.

Latent Heat

- Joseph Black (1728-1799) chemist, coined term latent heat.
  - 1760 - Studied phase transition of melting ice.
    $Q = mL$ for phase changes.
  - Measurements made 20 yrs later by Antoinne Laurent Lavoisier (1743-1794) and Pierre Simon Laplace (1749-1827).



Specific Heat Capacity

- 1779 - Adair Crawford (1748-1795) studied specific heat
  - oxygen involved in generation of heat in animals.

$$Q = mc\Delta T.$$

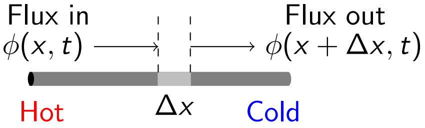
- 1783 - Lavoisier and Laplace, *Me'moir sur la Chaleur*, used ice calorimeter to obtain specific heats and latent heat (melting ice).
- Found that animals release heat - used guinea pig.

History of Heat Equation

Developed by Joseph Fourier (1768-1830)

- Discovered in early 1807 and published later in 1822
  - Afterwards, diffusion processes studied outside of France.
  - Lead to research in partial differential equations.
- Describes conduction and storage of heat (energy) in a body.
- Involves heat exchange with surroundings, conservation of energy.
- Leads to temperature changes inside body (diffusion).
- Uses the relation of heat energy to temperature (gradient), Fourier Law of Heat Conduction.

Heat Equation - Mathematics



Rate of change of heat energy = Flux in - Flux out

$$\frac{dQ}{dt} = \phi(x,t) - \phi(x+\Delta x,t)$$

Flux density = conductivity $\times$ temperature gradient

$$\phi = k\frac{dT}{dx}$$

Heat energy is proportional to temperature

$$Q = mcT$$

q = Heat energy per vol, u = temperature per vol

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}, \quad D = \frac{k}{mc}$$

Thanksgiving Turkey

- Native to North America
- Introduced in Spain in 1500's
- Benjamin Franklin - national bird
- Holiday bird in Europe in 1800's
  - \- replacing goose
- Turkeys mostly walk
- Harold McGee: Breast 155-160 F, Legs 180 F
- Cooking times
  - Constant oven temp, diffusivity constant, Turkey plump
  - Small - 20 min/lb + 20
  - Large - 15 min/lb + 15
  - $t \sim M^{2/3}$

How long does it take to cook a turkey?

Example 1

If it takes 4 hours to cook a 10 pound turkey in a 350^o F oven, then how long would it take to cook a 20 pound turkey at the same conditions?

Figure: A Thanksgiving turkey.

Panofsky Equation

- Pief Panofsky [SLAC Director Emeritus] *SLAC Today*, Nov 26, 2008
  `http://today.slac.stanford.edu/a/2008/11-26.htm`
  For a stuffed turkey at 325° F

  $$t = \frac{W^{2/3}}{1.5}$$

  vs. 30 minutes/lb.
- Also, check out WolframAlpha `http://www.wolframalpha.com/input/?i=how+long+should+you+cook+a+turkey`
- Musings of an Energy Nerd
  `http://www.greenbuildingadvisor.com/blogs/dept/musings/heat-transfer-when-roasting-turkey`

Consider a Spherical Turkey

Figure: The depiction of a spherical turkey.

Scaling a Spherically Symmetric Turkey

The baking follows the heat equation.
Rescale the coordinates (r, t) to (ρ, τ) :

$$r = \beta\rho \text{ and } t = \alpha\tau.$$

Then, the heat equation rescales as

$$u_\tau = \frac{\alpha}{\beta^2}\frac{k}{\rho^2}\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial u}{\partial\rho}\right).$$

- Invariance of heat equation implies $\alpha = \beta^2$.
- So, if the radius increases by a factor of β, then the time to cook the turkey increases by β^2.

Problem Solution

Example 1

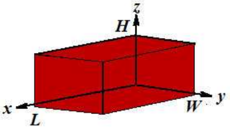
If it takes 4 hours to cook a 10 pound turkey in a 350^o F oven, then how long would it take to cook a 20 pound turkey at the same conditions?

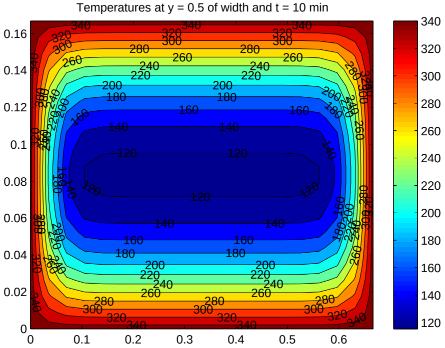
- The weight doubles $\Rightarrow$ the volume doubles.
  (if density = constant).
- $V \propto r^3 \Rightarrow r$ increases by factor: $2^{1/3}$.
- Therefore, the time increases by a factor of $2^{2/3} \approx 1.587$.
- If 4 lb turkey takes 4 hrs, then a 20 lb turkey takes

$$t = 4(2^{2/3}) = 2^{8/3} \approx 6.35 \text{ hours.}$$

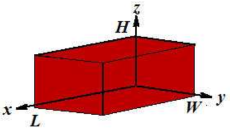
- In general, if the weight increases by a factor of x, then the time increases by $x^{2/3}$.

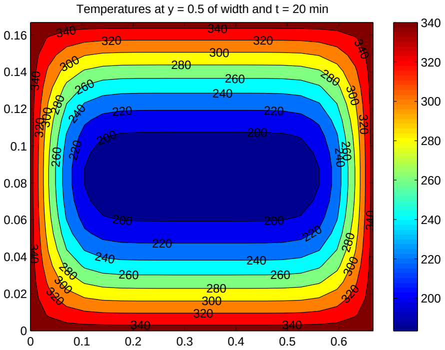
Baking a Rectangular Cake

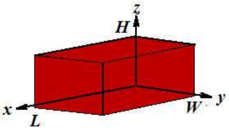


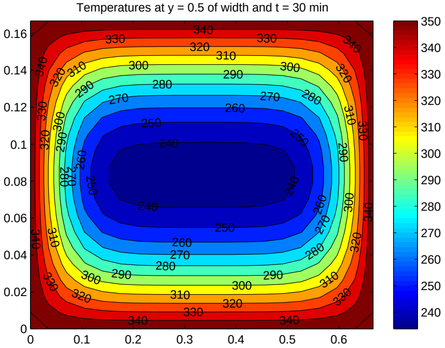


Baking a Rectangular Cake

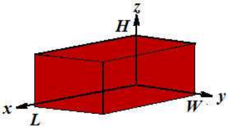


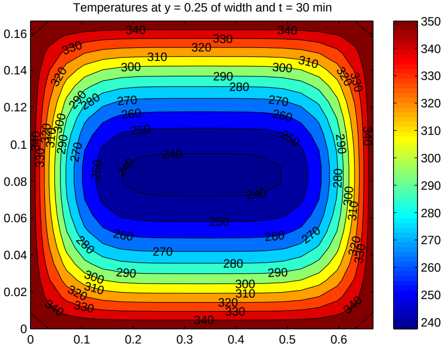



z
H
x
L
W
y

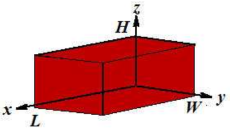

Temperatures at y = 0.5 of width and t = 30 min
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0 0.1 0.2 0.3 0.4 0.5 0.6
350
340
330
320
310
300
290
280
270
260
250
240

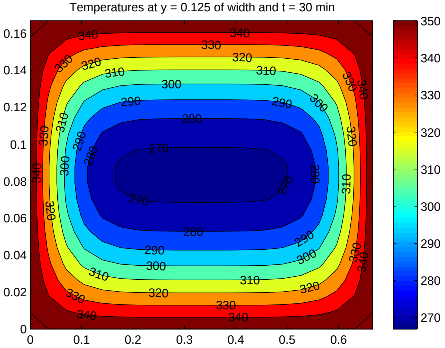
Baking a Rectangular Cake





Baking a Rectangular Cake





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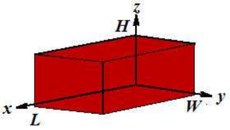
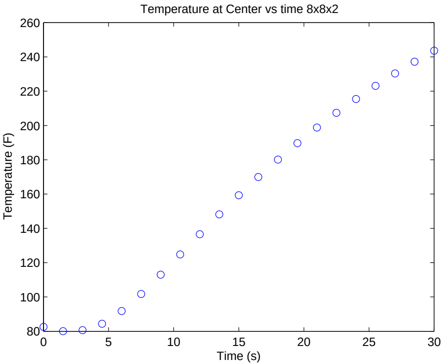
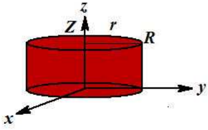
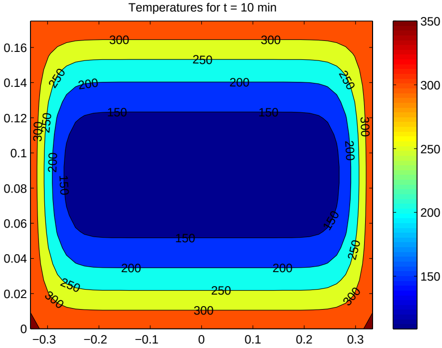


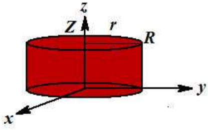


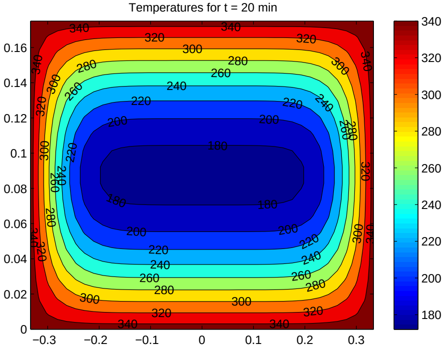
Baking a Circular Cake





Baking a Circular Cake





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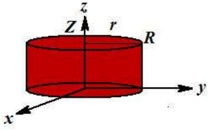

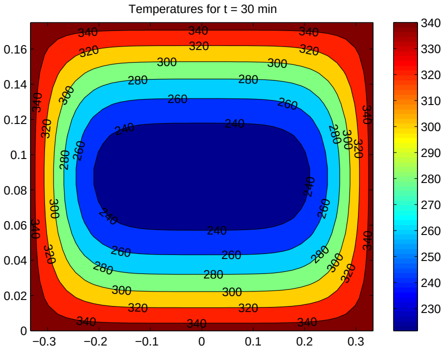

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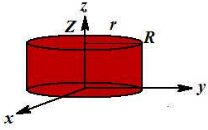
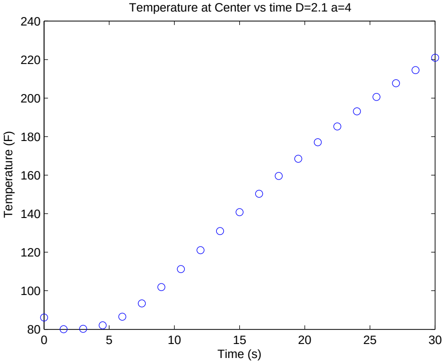
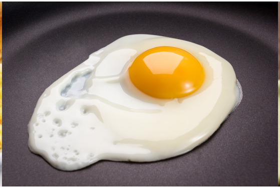




Eggs


Omelettes

Egg Protein

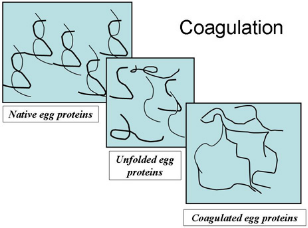
Proteins in eggs can be used

- to help food set (e.g. egg custards),
- as a foam to add air and volume (e.g. sponge cakes),
- as an emulsifier (e.g. mayonnaise).

Two different major proteins, egg white (albumin) and egg yolk,

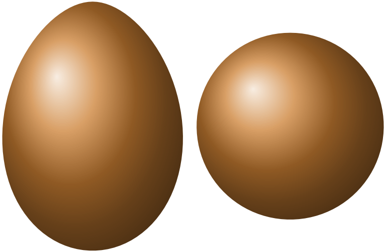
- Albumin starts coagulating at 63^oC
- Yolks start at 70^oC

Coagulation - protein unfolds, denaturation.
As heat increases the proteins rearrange and coagulate.
Egg albumin turns from clear to cloudy white.



Egg Cooking Time

Peter Barnham, *The Science of Cooking* & Dr. Charles Williams of Exeter:



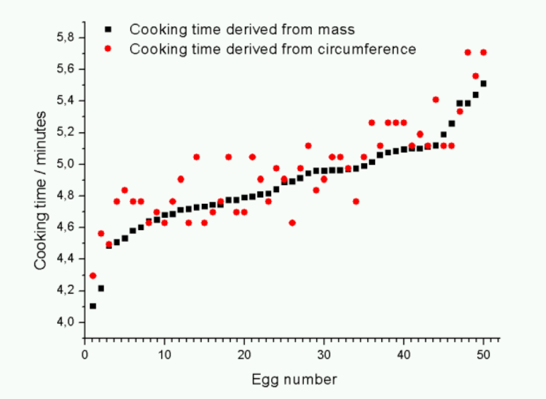
$$t = 0.0152 d^2 \log\left[2 \times \frac{(T_{\text{water}} - T_0)}{T_{\text{water}} - T_{\text{yolk}}}\right],$$

$$t = 0.451 M^{2/3} \log\left[0.76 \times \frac{(T_{\text{water}} - T_0)}{T_{\text{water}} - T_{\text{yolk}}}\right],$$

for t min, diameter d cm, M g, and temperatures in ^{o}C.

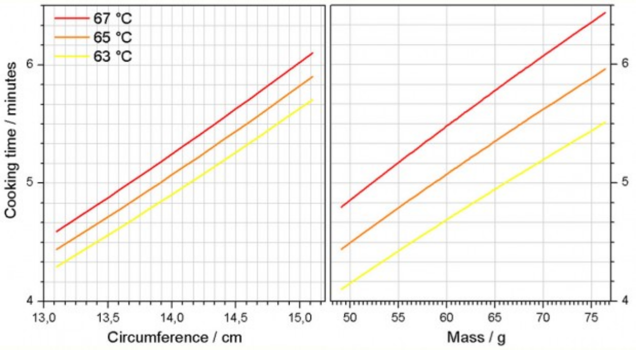
Egg Cooking Time - Data

From Khymos *Towards the perfect soft boiled egg* by Martin Lersch, April 9th, 2009. See also University of Oslo Applet



50 eggs with $T_{yolk} = 63^o$C, $T_{water} = 100^o$C and $T_{egg} = 4^o$C.

Egg Cooking Time - Formula



Given circumference or mass to reach to reach 63, 65 and 67^o C, respectively, at the yolk-white boundary with $T_{water} = 100^o$ C and $T_{egg} = 4^o$ C.

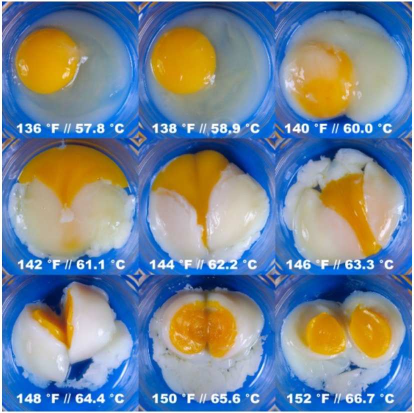
Egg Consistency

Temp	White	Yolk
62	Begins to set, runny	Liquid
64	Partly set, runny	Begins to set
66	Largely set, still runny	Soft solid
70	Tender solid	Soft solid, waxy
80	Firm	Firm
90	Rubbery solid	Crumbly texture

At sea level, boiling water is 100^o C. At higher altitudes, the boiling temperature of water is lowered 0.3^o C for each additional 100 m above sea level.

Sous Vide

Heat the yolk above 65^o C and the white above 80^o C. "Boil" the egg at a temperature lower than 100^o C. Eggs cooked sous vide for 75 min, Douglas Baldwin

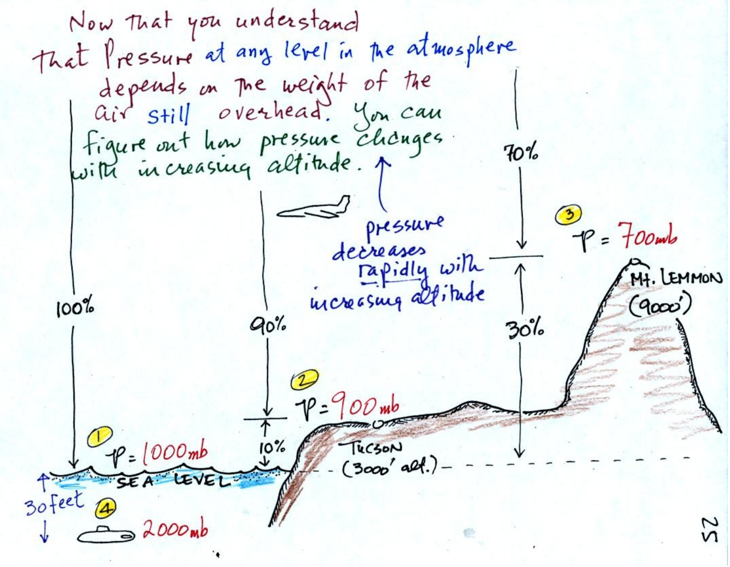


Summary

- Thermodynamics
  - Conduction
  - Convection
  - Radiation
  - Heat Equation
- Examples
  - Cooking Turkeys
  - Baking Cakes
  - Cooking Eggs

Thank You!
See MIT Lecture Series - Science and Cooking
`http://www.seas.harvard.edu/cooking`

Pressure



Now that you understand
that pressure at any level in the atmosphere
depends on the weight of the
air still overhead. You can
figure out how pressure changes
with increasing altitude.
70%
100%
90%
pressure
decreases
rapidly with
increasing altitude
3
P = 700mb
Mt. LEMMON
(9000')
30%
2
P = 900mb
1
P = 1000mb
10%
TUCSON
(3000' alt.)
SEA LEVEL
30 feet
4
2000mb