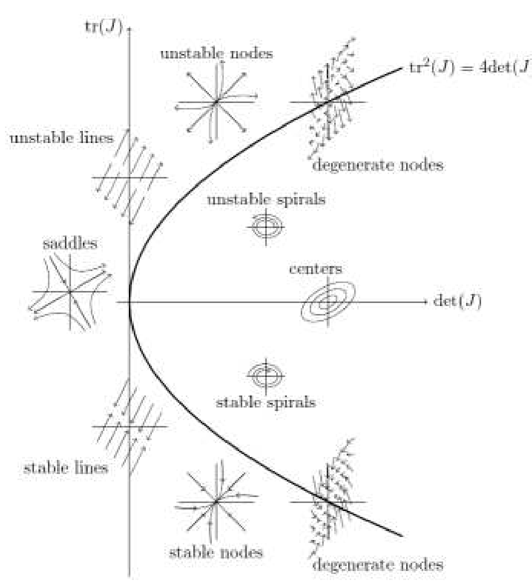
MAT 475/564 Exam II Review

1. Fractals
    a. Cantor set, Koch Curve
    b. Sierpinski triangle
    c. Mandelbrot and Julia sets
    d. Iterated function systems
    e. Linear and affine transformations
    f. Fractal dimension
    g. Ternary, binary, and other representations
2. Discrete Systems in the Plane
    a. Fixed points, periodic points
    b. Stability
    c. Chaos
    d. Lyapunov exponents and numbers
    e. Standard map
    f. Continued fractions
    g. Rotation numbers, Fibonacci numbers, golden mean
    h. KAM curves, invariant tori, quasiperiodic orbits
3. Linear Systems of Differential Equations
    a. Equilibrium Solutions
    b. Eigenvalue Problems – Solve for eigenvalues and eigenfunctions.
    c. Solution of systems – Use eigenvalues and eigenfunctions to construct solutions to systems. $\mathbf{v} = \begin{pmatrix} u \\ v \end{pmatrix} e^{\lambda t}$.
        i. Turn single ODEs into systems
        ii. Turn system into single ODE
        iii. Solve Second Order Linear Differential Equations of form $ax'' + bx' + cx = 0$
            1. Solve characteristic equation $a\lambda^2 + b\lambda + c = 0$.
            2. Three Cases:
                a. $x(t) = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t}$
                b. $x(t) = e^{\lambda t}(k_1 + k_2 t)$
                c. $x(t) = e^{\alpha t}[c_1 \cos(\beta t) + c_2 \sin(\beta t)]$, $\lambda = \alpha \pm i\beta$.
    d. Understand classification of Equilibrium Points and Connection to Phase Portraits, Eigenvalues and Solution Behavior.
    e. Nullclines
    f. Types: Stable/Unstable, Nodes, Foci, Centers, Degenerate Nodes, and Saddles.
    g. Stability Diagram



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4. Nonlinear Systems of Differential Equations
    a. Autonomous First Order Equations $\frac{dy}{dt} = f(y)$
        i. Equilibrium solutions $f(y_0) = 0.$
        ii. Classification (stable, unstable)
        iii. Phase Lines, Bifurcation Diagrams
            1. Saddle-Node Bifurcation
            2. Transcritical Bifurcation
            3. Pitchfork Birfurcation
    b. Nonlinear Systems
        i. Linearization About Equilibrium (Fixed) Points
        ii. Stability of Fixed Points
        iii. Identifying Interesting Features of Nonlinear Systems
    c. Special Systems
        i. Mass-spring $m\ddot{x} + b\dot{x} + kx = 0.$
        ii. Nonlinear Pendulum $m\ddot{x} + b\dot{x} + k\sin x = 0.$
        iii. Logistic Model $\dot{x} = ax(1-x)$