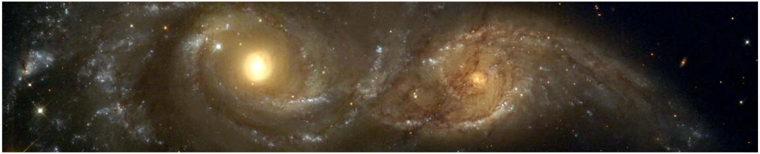

The Physics of Black Holes
PHY 490, Spring 2021

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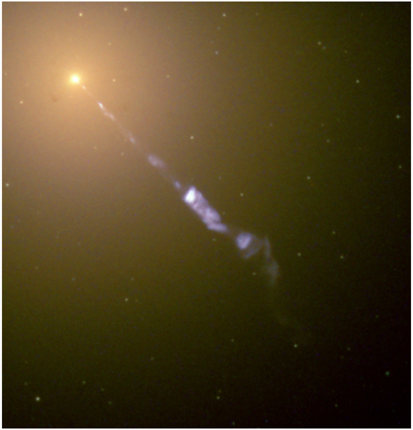


Figure 1: M87 Jet.

The Syllabus

- **Website**
  `http://people.uncw.edu/hermanr/BlackHoles/`
- **Grades**

Item	Percentage
Assignments	50%
Exams	40%
Paper	10%

- **Textbooks**

Black Holes: A Student Text, 3rd Edition, D. J. Raine and E. Thomas, 2015.

Gravity's Fatal Attraction (Black Holes in the Universe), 3rd Edition, M. Begelman and M. Rees, 2020.

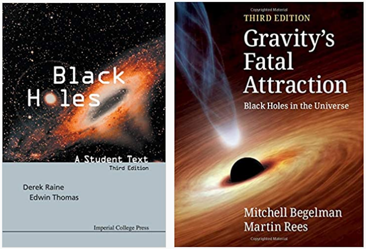
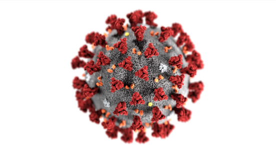


Figure 2: Main Textbooks.

COVID-19 Instruction

- This class is: Face to Face unless ...
- `https://uncw.edu/coronavirus/`
  - Social Distancing
  - Face Coverings
  - Wash Hands
- Office Hours



Following CDC Guidelines, UNC System directives, and out of mutual respect as outlined in the UNCW Seahawk Respect Compact, all faculty, staff, and students will wear face coverings while inside buildings. Students who are unprepared or unwilling to wear protective face coverings will not be permitted to participate in face-to-face sessions and will need to leave the building. Noncompliant students will be referred to the Dean of Students for an Honor Code Violation. Any student who has a medical concern with wearing a face covering should contact the Disability Resource Center at (910) 962-7555.

Students who experience COVID-19 symptoms should immediately contact the Abrons Student Health Center at (910) 962-3280.

Course Outline - *Black Holes: A Student Text*

- Compact Objects
- Special Relativity
- Vectors and Tensors
- General Relativity
- Schwarzschild Metric
    - Geodesics
    - Classic tests
    - Visualization
      Interstellar, EHT
- Kerr Metric
- Black Hole Thermodynamics
    - Information Paradox
- Wormholes and Time Travel

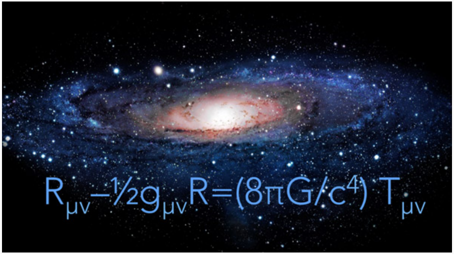
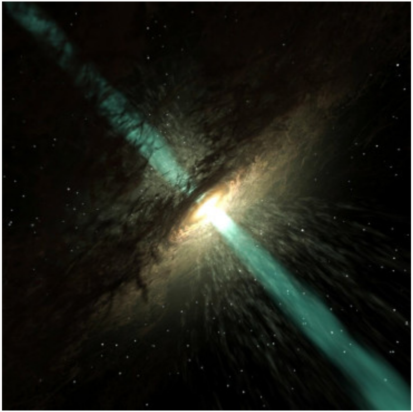


Figure 3: Einstein's Equation.

We begin with Chapter 7,
Astrophysical Black Holes,
and the book
Gravity's Fatal Attraction.

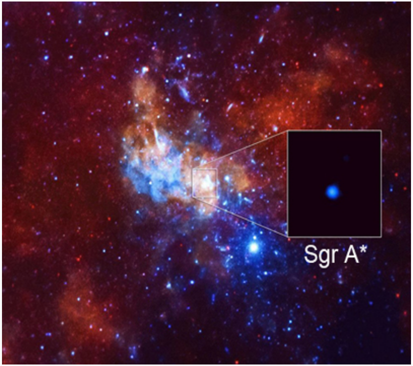
Course Outline - *Gravity's Fatal Attraction*

1. Gravity Triumphant
2. Stars and Their Fates
3. Black Holes in Our Backyard
4. Galaxies and Their Nuclei
5. Quasars and Kin
6. Jets
7. Blasts from the Past
8. Black Holes in Hibernation
9. Cosmic Feedback
10. Postcards from the Edge
11. Gravitational Waves
12. Through the Horizon



Compact Astrophysical Objects

- Endpoints of stellar evolution.
  - White dwarfs.
  - Neutron stars.
  - Black holes.
- Constituents of galaxies
- Extreme Objects at centers of
  - Milky Way - Sagittarius A* - $4.1 \times 10^6 M_\odot$,
  - M87* in Virgo Cluster - $6.5 \times 10^9 M_\odot$.
- Detection modes
  - Neutron stars, BHs - radio, X-ray emissions.
  - White dwarfs - optical.



https://www.nasa.gov/sites/default/files/chandra20140105.jpg

Black Holes

- 1783 - John Michell
  - applied gravity to corpuscules.
  - predicted dark stars.
- 1796 - Pierre-Simon Laplace
  - predicted point of no return.
- 1915 - Albert Einstein - GR.
- 1916 - Karl Schwarschild
  - Spherical symmetry.
- 1939 - J. Robert Oppenheimer and Hartland Snyder - Stellar collapse.
- 1939 - Einstein denied.
- 1967 - John Wheeler coined name.
- 1970 C.V. Vishveshwara.
  - Stability of Schwarzschild BH.
  - Quasinormal modes, ring down.

Figure 4: M87* - 2019 EHT Picture.

- 1964, 1971 - Cygnus X-1
  - 6070 lyr, $14.8 M_\odot$.
  - $R_s = 44$ km. [$1 M_\odot \rightarrow 2.95$ km]
- List of Black Holes

Black Holes Have No Hair

- Oppenheimer and Snyder assumed
  - Perfectly spherical.
  - Non-rotating.
  - No imperfections.
  - Controversial.
- 1964 - Roger Penrose
  If matter has a positive energy-density, a trapped surface has a singularity.
- 1966 - Stephen Hawking
  The Singularity Theorem is for the whole universe, and works backwards in time.
- 1967 - Werner Israel - 1^{st} Schwarzschild.
- 1972 - Jacob Bekenstein (via Wheeler)
  - "Black holes have no hair."
  - just mass, angular momentum, charge.

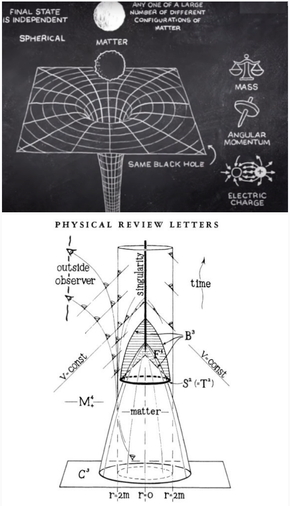


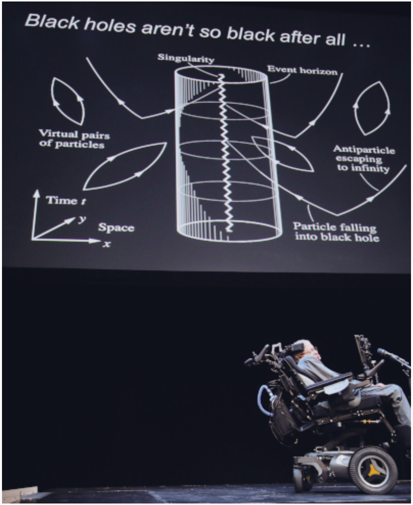
Figure 5: Video.

Black Hole Thermodynamics

- 1972 - Jacob Bekenstein
  Black holes should have entropy.
- 1974 - Stephen Hawking
  - They have a temperature.
  - And emit Hawking radiation.

$$S = \frac{kA}{4\ell_p^2}, \quad kT = \frac{hc^3}{16\pi^2 GM}.$$

- Laws of BH Thermodynamics.
- Black Holes Evaporate.
  Where does information go?
- Holographic Principle.
  - t'Hooft, Susskind.
  - Information Paradox.
  - AdS/CFT correspondence.



White Dwarfs

- 1783 -William Herschel.
  - 40 Eridani, triple system, 17 lyr
  - 1910 Henry Norris Russell, Edward Charles Pickering and Williamina Fleming identified as Spectral Class A.
- 1844 - Friedrich Wilhelm Bessel.
  - Sirius (Canus Major, Dog star, 8.6 lyr) and Procyon (Canus Minor, 12 lyr)
  - Companion white dwarfs.
- 1922 - Coined by Willem Luyten.
  - over 9000, 0.5-0.7$M_\odot$[0.8%-2% $R_\odot$].

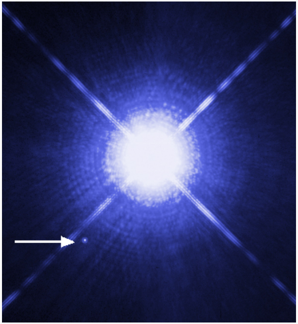


Figure 6: Sirius A and B.

https://en.wikipedia.org/wiki/White_dwarf

White Dwarf Stars Near The Earth

2018 - Astronomers Find Planet Vulcan Right Where Star Trek Predicted it.

Subrahmanyan Chandrasekhar (1910-1995)

- 1930 - At 19, traveled to England.
- Read William A. Fowler's 1926 e^--degeneracy.
  - In fermion gas, electrons move into unfilled energy levels.
  - Particle density increases and electrons fill the lower energy states.
  - Other e^-'s occupy states of higher energy (even at low temperatures).
  - Degenerate gases resist. compression due to the Pauli exclusion principle.
  - Generates a *degeneracy pressure*.
  - Applied Fermi-Dirac statistics.
- Degeneracy pressure vs gravity - **Chandrasekhar limit** - $M \sim 1.4 M_\odot$.

Neutron Stars

- 1932 - Chadwick discovers neutron.
- 1933 - Walter Baade, Fritz Zwicky - neutron stars result of supernovae.
- 1931,1937 - Lev Landau - Work on white dwarfs and neutron stars.
- 1939 - Oppenheimer-Volkoff-Tolman - max $M \sim 0.75 M_\odot$[Now, 1.5-3$M_\odot$].
- 1965 - Crab Pulsar, 1054 Supernova - Antony Hewish, Samuel Okoye.
- 1967 - Scorpius X-1, Iosif Schlovsky.
- 1967 - Jocelyn Bell, Antony Hewish - PSR B1919+21 pulsar.
- 1974 - Taylor-Hulse binary pulsar.

Figure 7: Crab Nebulae.

Taylor-Hulse Binary Pulsar PSR B1913+16

- Pulsars: pulsating radio star.
  Rapidly rotating neutron star.
- Magnetic lighthouse.
- Regular flashing
  - 2x each cycle - 17 per second.
- Regular variations - 7.75 hrs and 3s differences due to elliptical orbit.
- 305 m Arecibo Radio Telescope in Puerto Rico. (Collapse, Nov. 2020)
- 1993 Nobel Prize
  - Joe Taylor and Russell Hulse.

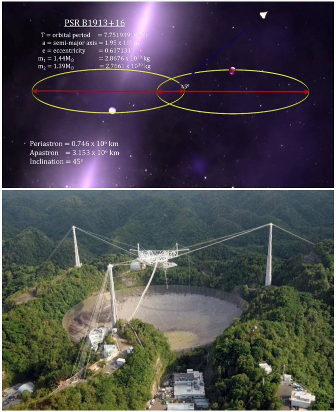


Figure 8: Binary Pulsar and Arecibo Telescope.

Binary Pulsar PSR B1913+16 and General Relativity

- Einstein's Prediction of radiation loss as gravitational waves.
- Calculated masses, periastron (closest distance), and apastron (furthest).
- Energy Loss: $\dfrac{dE}{dt} = 7.35 \times 10^{24}\text{W}$.
- Orbital period change: $\dfrac{dT}{dt} = 7.65$ milliseconds/yr.
- First indirect observation of gravitational waves.

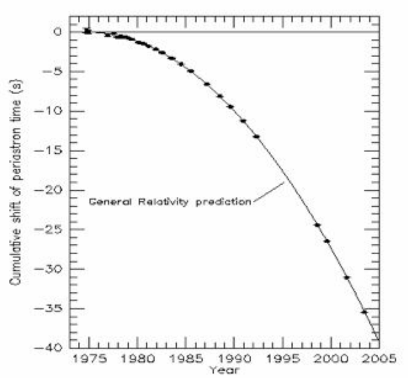
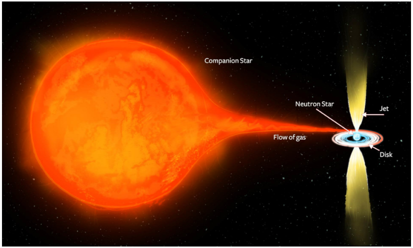


Figure 9: Binary Pulsar Data

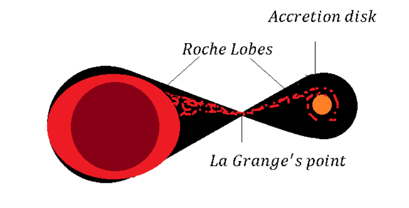
Accretion in Binary Systems

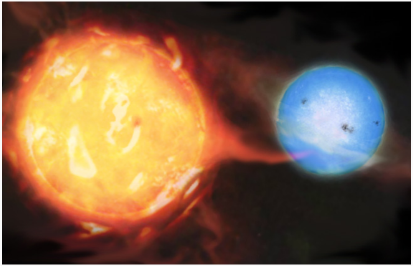
- 1960s - X-Ray Astronomy.
- Scorpius X-1: 1-10 keV.
- 20 sources by end of decade.
- Cygnus X-1 varies in time.
- Accretion.
    - Gas forms disk around compact object.
    - Friction leads to spiraling inward.
    - Gravity and friction compress, raise temperature.
    - Leads to EM emission.



Accretion History

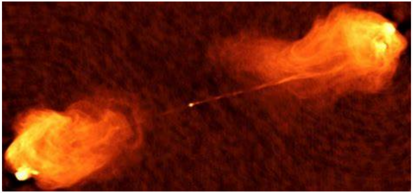
- 1926 Arthur Eddington -
  accretion rate depends on
  velocity, density
  gravity focuses towards CM.
- Hoyle, Lyttleton - rate greater with collisions.
- 1952 Herman Bondi
- Algol in Perseus
  - eclipsing binary, 2.9 days.
- Small blue hidden by larger red.
- Blue star tending to red giant.
- Used to be red, overflowed Roche lobe - past Lagrange pt.





Active Galaxy Nuclei

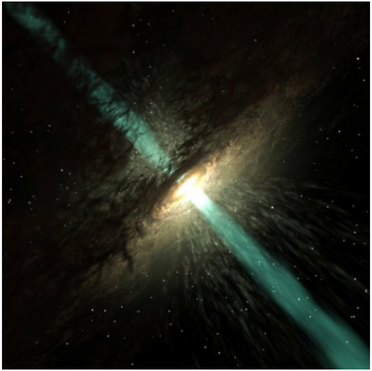
- 1918 - Heber Curtis
  - Straight ray from M87.
- 1920 - Island Universes
  - Curtis-Shapley Debate
- Strange behavior from galaxy centers.
  - Too much blue, UV.
  - Bright.
  - Active galaxies with AGN.
  - Quasars, starbursts.
- Karl Seyfert
  - Intense blue nuclei.
  - Very high velocities.
  - Seyfert galaxies.



- 1954 - W. Baade, R. Minkowski
  - Cynus A
  - 300 $\times$ M31 distance.
  - Dumbbell - lobes.
- 1956-9 - Geoffrey Burbidge
  Lobe energy very high.

Quasars - Quasi Stellar Objects

- 1963 - Hazard, et al. pinpoint 3C 273.
- 1963 - Maartin Schmidt, 3C 273.
  - Spectrum: 16 % redshift
  - Distance: 10^9 lyr.
  - Fluctuating brightness over 1 mo.
- Quasars - QSOs.
- Hubble Telescope detects many.
- AGN Properties:
  - Energy emitted large rate.
  - Extremely compact.
  - Not normal radiation.
  - Gas moves at very high speeds.
- Manifestation of massive BHs.



Massive Black Holes?

- Mass Compactness, M/R, Limits
  - Upper limit on R
    - brightness variation. $R < ct$.
  - Lower limit on M - luminosity.
  - Estimate lifetime energy.
    - luminosity $\times$ age.
    - Eddington limit.
    - 100 million to billions $M_\odot$.
  - $M/R > .001c^2/G$.
- Hoyle, Burbidge - only gravitational collapse can supply energy.

Quasar Models

- 1969 - Donald Lynden-Bell
  - Quasars powered by accretion.
  - BHs > 100 million $M_{\odot}$.
- Sources of Emission
  - Accretion disk like X-Ray binaries.
  - EM processes
    - Tap spin energy.
    - A flywheel with disk as brake.
- Different emissions
  - X-Rays captured by disk.
  - Turned to UV, optical, IR.
- Need Mass, Spin, and Orientation.

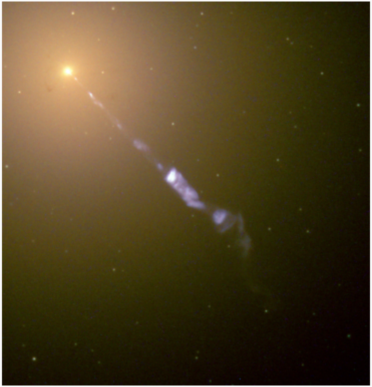


Figure 10: M87 jet.

Radio Astronomy

- 1931 - Karl Jansky
  - Bell Labs - telecommunications.
  - Sensitive antenna
    - transaltantic cable noise.
    - Not terrestrial!.
- 1944 - Grote Reber
  - First sky map of Milky Way.
  - Radio Emissions, Cygnus.
- 1950s - Martin Ryle
  - Idea of arrays of dishes.
- 1970s - Telescope arrays.
- Detailed hot spots and lobes.

VLA - Very Large Array
- Socorro, NM.
27 linked radio telescopes.
25 m diameter.
Y-shaped across 40 km.
Comparable to Hubble resolution.

Hot Spots and Lobes

- Picked up double radio sources.
- From galaxy cores.
- Superhot, magnetized gas ejection?
- 70s - Blobs powered by twin streams of gas from galactic core hotspot.
  - Source of radio waves.
- Travels through intergalactic medium, pushing matter away at 60% c.
- Deceleration leads to shock waves.
- Energy of relativistic e^-'s and magnetism. - Synchrotron radiation.
- Hot spot - 100,000 to 10^6 yrs.
- Moves to lobes, persists 10^8 yrs.

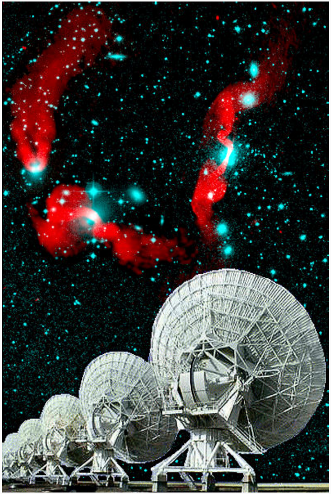
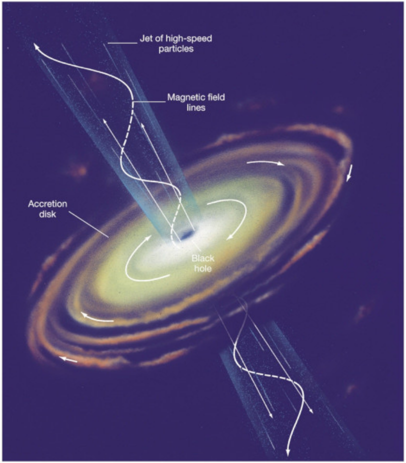


Figure 11: Radio Galaxies and Quasars

Jets

- 1978 - Jets only theoretical.
- Not seen to this point.
- VLA observations changed that.
- Late 1978 - SS 443 X-ray binary.
- Bruce Margon, et al.
  - Spectrum had three parts:
  Normal, red shifted, blue shifted.
- Rotating jets precessing - 163 days.
- Stability due to high speed, low density, stiff B-field.
- Can lead to radio trail bend.
- Then there is the core using VLBI.



Black Holes - The Last Decade

- 2014- *Interstellar,* the movie.
  - Black hole visualization.
- 2016 - Gravitational Waves, LIGO.
- 2019 - First picture, Event Horizon.
- Nobel Prizes:
  - 2011 - Saul Perlmutter, Brian P. Schmidt and Adam G. Riess.
  - 2017 - Rainer Weiss, Barry C. Barish and Kip S. Thorne.
  - 2019 - James Peebles, Michel Mayor and Didier Queloz.
  - 2020 - Roger Penrose, Reinhard Genzel and Andrea Ghez
- Now, back to physics ...

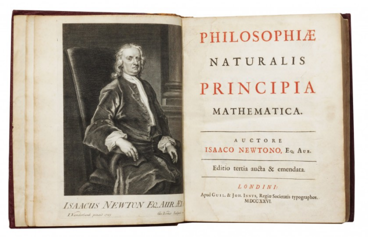
Isaac Newton (1642-1727)

In 1680s Newton sought derivation of Kepler's planetary laws of motion.

- Principia 1687.
- Took 18 months.
- Laws of Motion.
- Law of Gravitation.
- 1759 - Halley's Comet

Objects on the Earth feel same force as the planets orbiting the sun.

$$F = G\frac{mM}{r^2}.$$



John Michell (1724-1793) - restored from obscurity

- Natural philosopher, clergyman
- Applied Newton's Corpuscular Theory.
- *Philosophical Transactions of the Royal Society of London*, 1783.
- A star's gravitational pull might be so strong that the escape velocity would exceed the speed of light!
  - **Dark Stars**.
- Pierre-Simon Laplace (1749-1827), *Exposition du Système du Monde* - 1796
- Consider **escape velocity**.

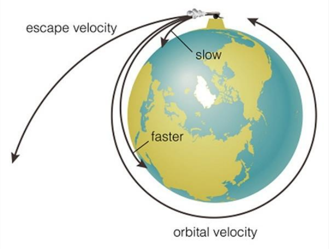


Figure 12: Firing projectiles.

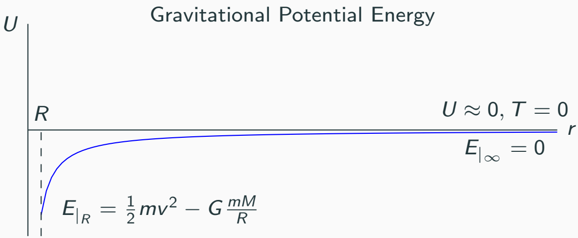
Escape Velocity from $E = T + U$

- Kinetic energy: $T = \frac{1}{2}mv^2$.
- Potential energy:

$$\begin{aligned} U &= \int_\infty^R F(\rho)\, d\rho \\ &= \int_\infty^R G\frac{mM}{\rho^2}\, d\rho = -G\frac{mM}{R}. \end{aligned}$$

- Escape velocity:
  Energy conservation.

$$\begin{aligned} \frac{1}{2}mv^2 - G\frac{mM}{R} &= 0. \\ v &= \sqrt{\frac{2GM}{R}}. \end{aligned}$$



Common Escape Velocities, $v = \sqrt{\frac{2GM}{R}}$,

Escape rates for some celestial bodies, $G = 6.67 \times 10^{-11} Nm^2/kg^2$.

	Mass M (kg)	Radius R (m)	Escape Velocity v (m/s)
Moon	7.348×10^{22}	1.737×10^{6}	2,376 (5,300 mph)
Earth	5.972×10^{24}	6.378×10^{6}	11,176 (25,000 mph)
Jupiter	1.898×10^{27}	7.1492×10^{7}	59,511 (133,000 mph)
Sun	1.989×10^{30}	6.957×10^{8}	617,567 (1.38 million mph)

For light, $R = \frac{2GM}{c^2}$,
$v = c = 3.0 \times 10^8$ m/s.

- Earth, $R = .0088$ m.
- Sun, $R = 2.9$ km,
- $\frac{\text{Sun Mass}}{\text{Earth Mass}} = 3.3 \times 10^5$

But, light is a wave!

Page 1 of text: Let $\rho \sim M/R^3$. Light fails to escape when

$$M \sim (c^2/G)^{3/2} \rho^{-1/2}$$

For lead, $\rho \sim 5000$ kg-m^{-3},
$M \sim 7.01 \times 10^{38} kg = 3.5 \times 10^8 M_\odot$.
Then explain Eq. (1.1),

$$M \sim 10^8 (\rho_*/\rho)^{1/2} M_\odot.$$

James Clerk Maxwell (1831-1879) - Light = EM Wave

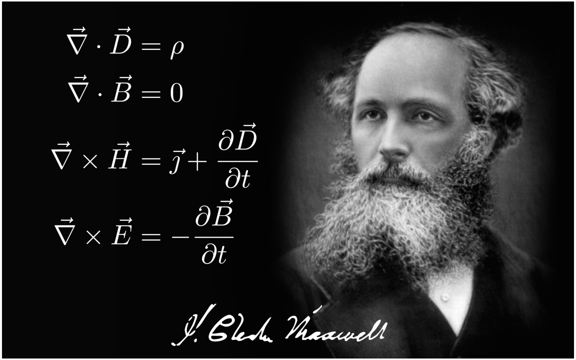


Figure 13: Equations of Electricity and Magnetism
Gauss' Law, No magnetic monopoles, Maxwell-Ampere Law, Faraday's Law.

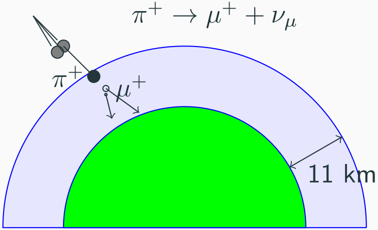
1905 - Einstein's Miracle Year

- Photoelectric effect (March/June).
- Brownian motion (May/July).
- Special Relativity (June/September).
  - Inspired by Maxwell's Theory.
  - Two Postulates
    - Physics is same for all inertial observers.
    - Speed of light same for everyone.
  - Consequences.
    - Time dilation.
    - Length contraction.
    - Space and Time relative.
- $E = mc^2.(September/November)$

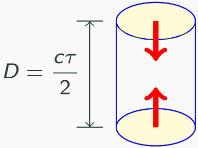
Figure 14: Einstein (1879-1955)

Time Dilation - Moving clocks tick slower.

- Examples -
  - Plane trip
    - 620 mph (277 m/s)
    - Lose 3 ns/hr.
  - Muon
  - Cosmic rays collide with nuclei.
  - Pions decay into muons.
    - Lifetime 2.2 μs
    - At 0.995c, travels 660 m

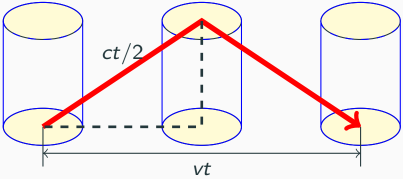


Light clock



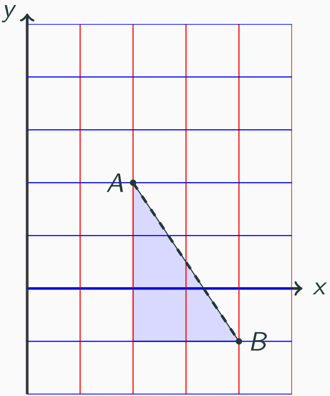
Moving light clock

$$t = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

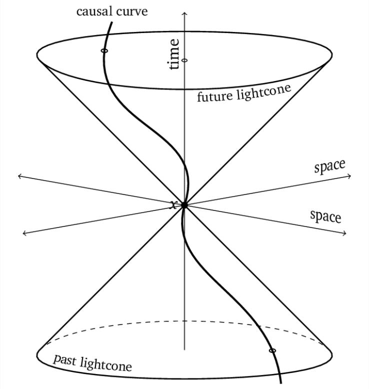


Space, Time, and Spacetime

From René Descartes:

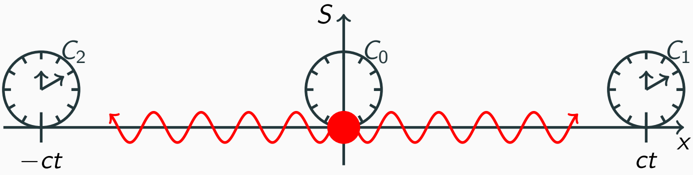


From Hermann Minkowski:

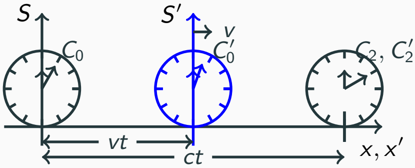


Particles move in straight lines to maximize lifetime.

Lorentz Transformation



- Clock C_0, synchronized with C_1, C_2.
- Pulse sent at at $t = 0$.
- Travels ct to C_1, C_2.
- Then, $x = \pm ct$, or $x^2 - c^2t^2 = 0$.
- System S' travels v w.r.t. S.
- $x'^2 - c^2t'^2 = 0$.
- $\Delta x^2 - c^2\Delta t^2 = \Delta x'^2 - c^2\Delta t'^2$.

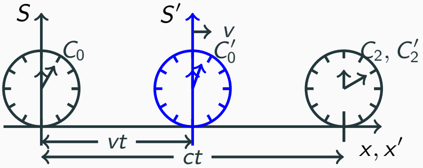


Lorentz Transformation (con't)

- $\Delta x^2 - c^2\Delta t^2 = \Delta x'^2 - c^2\Delta t'^2$.
- $\Delta x' = 0$, C_0' at rest w.r.t. S'.
- According to S, C_0' at $x = vt$.

$$\begin{aligned} x^2 - c^2t^2 &= -c^2t'^2 \\ (v^2 - c^2)t^2 &= -c^2t'^2 \\ t &= \gamma t' \\ \gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}. \end{aligned}$$

- Galilean transformation, $x = x' + vt', t = t'$.
- Assume Lorentz transformation, $x = ax' + bct', t = \gamma t'$.



- $x' = 0, x = vt \Rightarrow vt = bc\gamma^{-1}t$. So, $b = \beta\gamma$, $\beta = v/c$.
- $x = 0, x' = -vt \Rightarrow$ $0 = -avt' + bct'$, or $a = \gamma$.
- Thus, $x = \gamma\left(x' + \beta ct'\right).$
- $t = x/c, t' = x'/c \Rightarrow$ $ct = \gamma(ct' + \beta x')$.

Lorentz Transformation Summary

The Lorentz transformation in 1+1 dimensional spacetime is

$$
\begin{aligned}
x &= \gamma(x' + vt') = \gamma(x' + \beta ct'), && (1)\\
ct &= c\gamma(t' + \frac{vx'}{c^2}) = \gamma(ct' + \beta x'), && (2)
\end{aligned}
$$

with Lorentz factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$.

The inverse transformation is

$$
\begin{aligned}
x' &= \gamma(x - vt) = \gamma(x - \beta ct), && (3)\\
ct' &= c\gamma(t - \frac{vx}{c^2}) = \gamma(ct - \beta x). && (4)
\end{aligned}
$$

This is also referred to as a Lorentz boost.

Matrix Representation

$$\begin{aligned}\begin{pmatrix} x \\ ct \end{pmatrix} &= \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} x' \\ ct' \end{pmatrix} \\ &= \begin{pmatrix} \cosh\chi & \sinh\chi \\ \sinh\chi & \cosh\chi \end{pmatrix}\begin{pmatrix} x' \\ ct' \end{pmatrix}\end{aligned}$$

Here $\beta = \tanh\chi$, $\gamma = (1-\beta^2)^{-1/2} = \cosh\chi$, where χ is called the rapidity. The inverse transformation is given by

$$\begin{aligned}\begin{pmatrix} x' \\ ct' \end{pmatrix} &= \begin{pmatrix} \cosh\chi & \sinh\chi \\ \sinh\chi & \cosh\chi \end{pmatrix}^{-1}\begin{pmatrix} x \\ ct \end{pmatrix} \\ &= \begin{pmatrix} \cosh\chi & -\sinh\chi \\ -\sinh\chi & \cosh\chi \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix}\end{aligned}$$

Group Structure of Lorentz Boost

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\chi & -\sinh\chi \\ -\sinh\chi & \cosh\chi \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \equiv \Lambda(\chi) \begin{pmatrix} x \\ ct \end{pmatrix}.$$

- Composition $\Lambda(\chi_1)\Lambda(\chi_2) = \Lambda(\chi_1 + \chi_2)$.

$$\begin{pmatrix} \cosh\chi_1 & -\sinh\chi_1 \\ -\sinh\chi_1 & \cosh\chi_1 \end{pmatrix} \begin{pmatrix} \cosh\chi_2 & -\sinh\chi_2 \\ -\sinh\chi_2 & \cosh\chi_2 \end{pmatrix} = \begin{pmatrix} \cosh(\chi_1+\chi_2) & -\sinh(\chi_1+\chi_2) \\ -\sinh(\chi_1+\chi_2) & \cosh(\chi_1+\chi_2) \end{pmatrix}$$

- Addition of Velocities:

$$\begin{aligned} \tanh\chi &= \frac{\tanh\chi_1 + \tanh\chi_2}{1 + \tanh\chi_1 \tanh\chi_2} \\ v &= \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}. \end{aligned}$$

- Identity ($\chi = 0.$), Inverse, Associative.
- Similar to (imaginary) rotation group.

Addition of Velocities

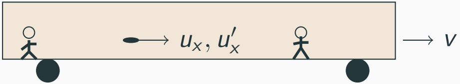
A passenger fires a bullet at 0.6c relative to a train moving at 0.8c. How fast is the bullet moving relative to the ground? It is not 1.4c.

Another derivation:

$$\begin{aligned} dx &= \gamma(dx' + \beta c dt') &= \gamma(u_x' + v)dt', \\ dt &= \gamma\left(dt' + \frac{v}{c^2}dx'\right) &= \gamma\left(1 + \frac{vu_x'}{c^2}\right)dt'. \end{aligned}$$

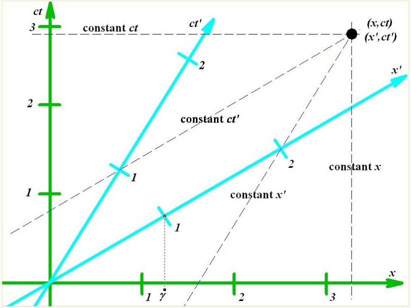
So,

$$u_x = \frac{dx}{dt} = \frac{u_x' + v}{1 + \frac{vu_x'}{c^2}}$$



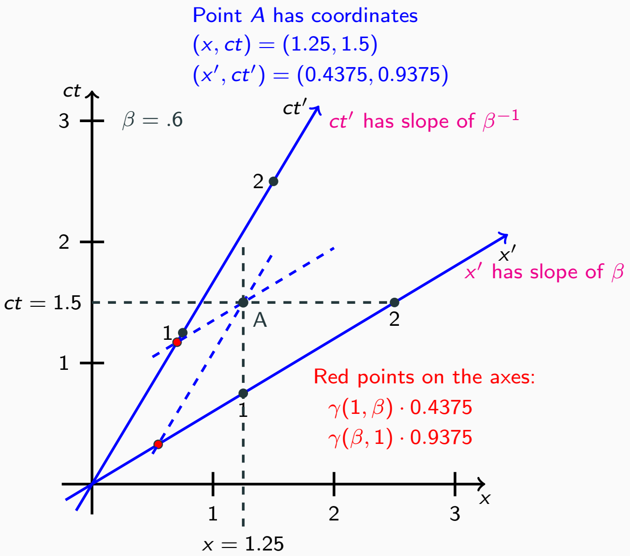
Minkowski Diagrams

- Reference frame $S : (x, ct)$.
- Reference frame $S' : (x', ct')$.
- x'-axis: $x' = 1$, $ct' = 0$.
- Then, $x = \gamma$, $ct = \beta\gamma$.
- Thus, $ct = \beta x$.
- x'-axis has slope $\beta = v/c$
- ct' axis: $x' = 0$, $ct' = 1$.
- Then, $ct = \gamma$, $x = \gamma\beta = \beta ct$.
- Thus, ct'-axis has slope $1/\beta = c/v$.

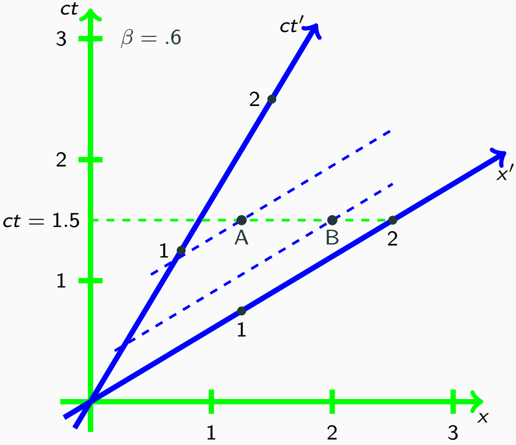


In Figure $\beta = 0.6$. Thus, $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{5}{4}$.
From $x = \gamma$ and $ct = \beta\gamma$, locate the $(1, 0)$ in the primed system.

Reading Coordinates on a Minkowski Diagram

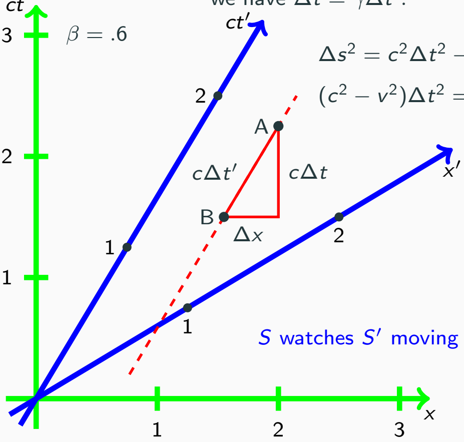


Simultaneity



Time Dilation

Using $c\Delta t = \gamma(c\Delta t' + \beta\Delta x')$ and $\Delta x' = 0$,
we have $\Delta t = \gamma\Delta t'$.



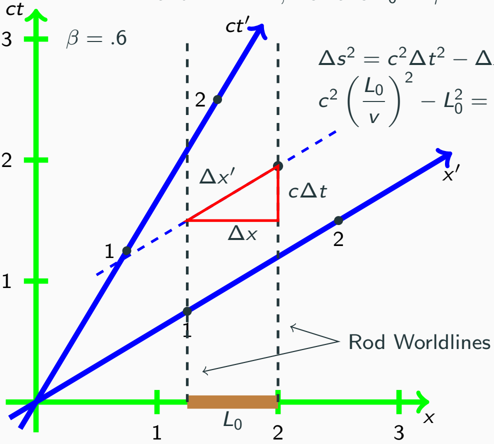
$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2$$

$$(c^2 - v^2)\Delta t^2 = c^2\Delta t'^2 \Rightarrow \Delta t = \gamma\Delta t'.$$

S watches *S'* moving from *A* to *B*.

S' looks at a rod at rest in the S-frame.

Using $\Delta x = \gamma(\Delta x' + \beta c \Delta t')$, $c\Delta t' = 0$, $\Delta x = L_0$ and $\Delta x' = L$, we have $L_0 = \gamma L$.

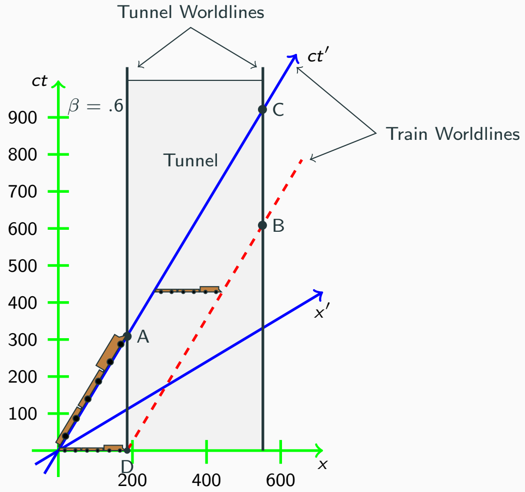


$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 = -\Delta x'^2$$

$$c^2 \left(\frac{L_0}{v}\right)^2 - L_0^2 = -L^2 \Rightarrow L_0 = \gamma L.$$

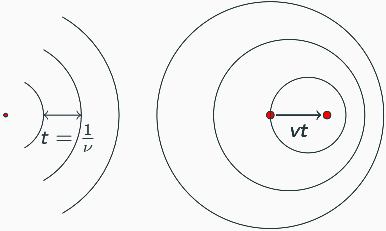
Train in Tunnel Problem

A relativistic train of rest length 240 meters travels at 0.6c through a tunnel which has rest length 360 meters.



Doppler Effect for a Moving Source

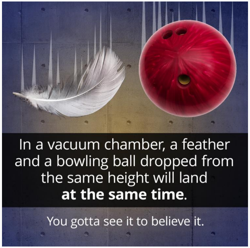
- Classical Doppler: $\lambda' = \frac{c}{\nu} - vt = \frac{c}{\nu}(1-\beta)$.
  Apparent frequency: $\nu' = \frac{c}{\lambda'} = \frac{\nu}{1-\beta}$.

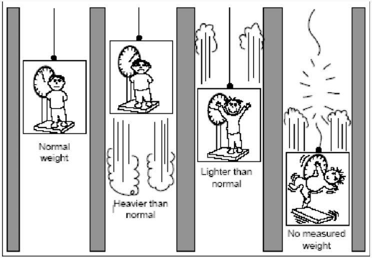


- Relativistic Doppler: Source clock ticks slower, $\nu \to \nu/\gamma$.
  Apparent frequency: $\nu' = \frac{\nu}{\gamma(1-\beta)} = \nu\sqrt{\frac{1+\beta}{1-\beta}}$.
- Galaxy moves away ($\beta < 0$) - **redshift** ($\nu' < \nu$ and $\lambda' > \lambda$).

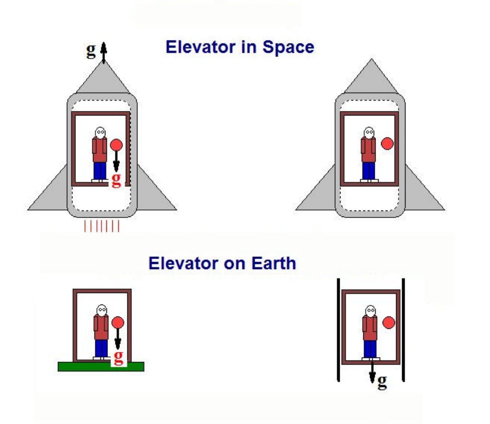
Einstein's Happiest Thought

- Einstein spent years generalizing Special Relativity.
- Galileo - Everything falls at the same rate.
- Einstein - When you fall freely, gravity disappears.
- Led to the Equivalence Principle.





The Equivalence Principle



There are no (local) experiments which can distinguish non-rotating free fall under gravity from uniform motion in space in the absence of gravity.

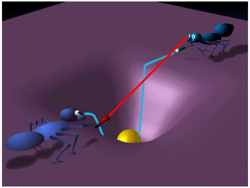
General Relativity - 1915

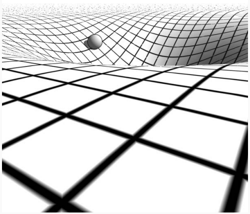
Einstein generalized special relativity to Curved Spacetime.

- Einstein's Equation.
- Gravity = Geometry

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$

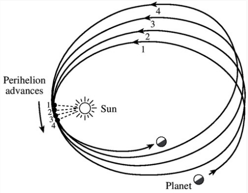
- Mass tells space how to bend and space tell mass how to move.
- Predictions. (Wheeler)
  - Perihelion Shift of Mercury.
  - Bending of Light.
  - Time dilation.





Classical Tests - Perihelion Shift of Mercury

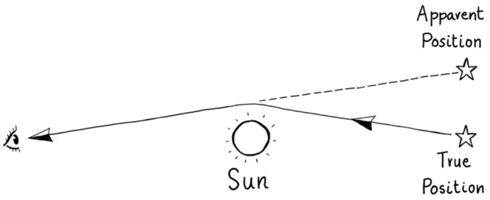
- First noted by Le Verrier, 1859. $38''$ (arc seconds) per century.
- Re-estimated by Newcomb, 1882.
- Ellipse axis shifts $43''$ per century.



arcsec/cent	Cause
532.3035	Gravitational tugs by other bodies
0.0286	Oblateness of Sun
42.9799	General Relativity
-0.0020	Lense-Thirring
575.31	Total Predicted
574.10 ± 0.65	Observed

Classical Tests - Deflection of Light

- Deflection of light - when light passes near a large mass its path is slightly bent.
- 1919 Eclipse observed an island near Brazil and near the west coast of Africa.

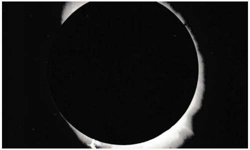


LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

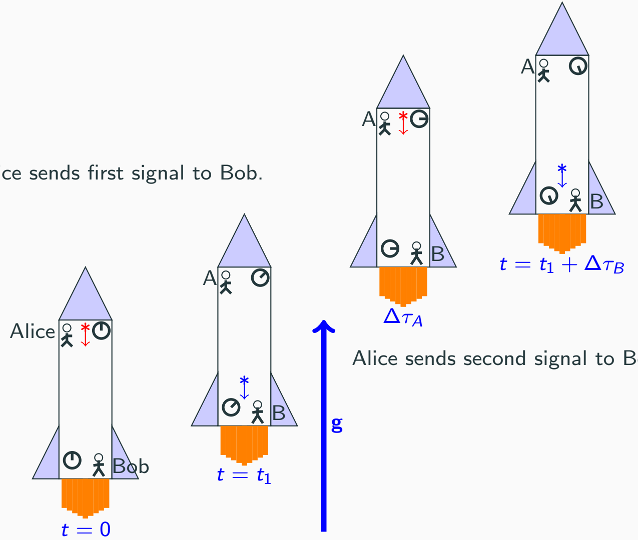
EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.



Classical Tests - Gravitational Time Dilation

Alice sends first signal to Bob.



Alice sends second signal to Bob.

Derivation of Gravitational Time Dilation

- Bob and Alice's positions for accelerating rocket:
  $z_B(t) = \frac{1}{2}gt^2, \qquad z_A(t) = h + \frac{1}{2}gt^2.$
- Pulse emitted at $t = 0$ and received at t_1 : $z_A(0) - z_B(t_1) = ct_1$.
- Second pulse emitted travels distance
  $z_A(\Delta\tau_A) - z_B(t_1 + \Delta\tau_B) = c(t_1 + \Delta\tau_B - \Delta\tau_A)$
- Assume $\Delta\tau_A$ small, we have

$$\begin{aligned} h - \frac{1}{2}gt_1^2 &= ct_1, \\ h - \frac{1}{2}gt_1^2 - gt_1\Delta\tau_B &= c(t_1 + \Delta\tau_B - \Delta\tau_A). \end{aligned} \tag{5}$$

- Assume gh/c^2 small, $t_1 \approx h/c$ and

$$\Delta\tau_B = \Delta\tau_A \left(1 - \frac{gh}{c^2}\right).$$

Gravitational Redshift

The time interval for received pulses is smaller

$$\Delta\tau_B = \Delta\tau_A\left(1 - \frac{gh}{c^2}\right).$$

In general, note $gh = \Phi_A - \Phi_B$ is gravitational potential difference. Then, the rate of emission and reception, $1/\Delta\tau$, is

$$\omega_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right)^{-1}\omega_A \approx \left(1 + \frac{\Phi_A - \Phi_B}{c^2}\right)\omega_A$$

For a star of radius R and signal received far away, and noting $\Phi_A - \Phi_B = \frac{GM}{r_B} - \frac{GM}{r_A}$, we have the **gravitational redshift**

$$\omega_\infty = \left(1 - \frac{GM}{Rc^2}\right)\omega_{\text{star}}.$$

Time Dilation and GPS

Gravitational redshift - clocks in a gravitational field observed from a distance tick slower. (1960s, Pound-Rebka-Snider experiments)

- Special Relativity.

$$\delta t = \frac{\delta \tau}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

- General Relativity.

$$\begin{aligned} \delta t &= \delta \tau \sqrt{1 - \frac{2GM}{rc^2}} \\ &\approx \delta \tau \left(1 - \frac{GM}{rc^2}\right). \end{aligned}$$

- Application - GPS

GPS Satellites

- Global Positioning System
- 32 Satellites (max)
- Semi-synchronous orbits
  - 20,200 km,
  - 11 hours 58 min
  - Cesium or Rubidium clocks
- At least 4 over each location
- SR: Lose 7,200 ns/day
- GR: Gain 45850 ns/day
- Net, 39 μs/day [or, 500 m/hr]

Triangulation

Equations of intersecting circles:

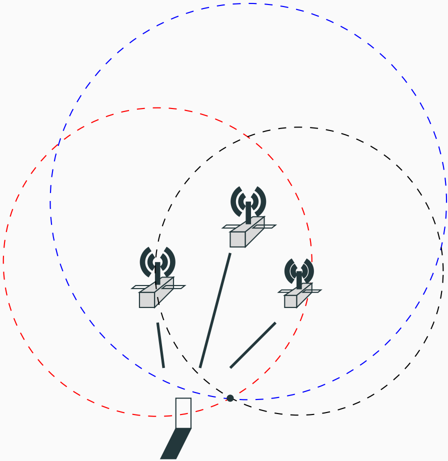
$$\begin{aligned}(x-14)^2+(y-45)^2 &= 39^2.\\(x-80)^2+(y-70)^2 &= 50^2.\\(x-71)^2+(y-50)^2 &= 29^2.\end{aligned}$$

Subtract first and last pairs:

$$\begin{aligned}132x+50y &= 8100,\\18x+40y &= 2100.\end{aligned}$$

Solve: $x=50, y=30$.

For satellites, use intersecting spheres and vertical coordinate, z.



Newtonian Gravity and Spacetime

Consider the line element

$$ds^2 = -\left(1 + \frac{2\Phi(x^i)}{c^2}\right)(cdt)^2 + \left(1 + \frac{2\Phi(x^i)}{c^2}\right)^{-1}(dx^2 + dy^2 + dz^2).$$

Then, the proper time between points A and B is

$$\begin{aligned}
\tau_{AB} &= \int_A^B d\tau = \int_A^B \left(\frac{ds^2}{c^2}\right)^{1/2} \\
&= \int_A^B \left[\left(1 + \frac{2\Phi(x^i)}{c^2}\right) dt^2 - \frac{1}{c^2}\left(1 + \frac{2\Phi(x^i)}{c^2}\right)^{-1}(dx^2 + dy^2 + dz^2)\right]^{1/2} \\
&= \int_A^B dt \left[\left(1 + \frac{2\Phi(x^i)}{c^2}\right) - \frac{1}{c^2}\left(1 + \frac{2\Phi(x^i)}{c^2}\right)^{-1} v^2\right]^{1/2} \\
&\approx \int_A^B dt \left[1 + \frac{2\Phi(x^i)}{c^2} - \frac{1}{c^2} v^2\right]^{1/2} \approx \int_A^B dt \left[1 + \frac{1}{c^2}\left(\Phi(x^i) - \frac{1}{2}v^2\right)\right]
\end{aligned}$$

Path of Extremal Proper Time

The proper time between points A and B to first order in $1/c^2$ is

$$\tau_{AB} = \int_A^B dt \left[1 + \frac{1}{c^2}\left(\Phi(x^i) - \frac{1}{2}v^2\right)\right]$$

Extremizing is equivalent to extremizing

$$I = \int_A^B dt \left(\frac{1}{2}v^2 - \Phi(x^i)\right).$$

We have the Lagrangian $L = \frac{1}{2}v^2 - \Phi(x^i)$. The Lagrange equations give

$$\frac{d^2\mathrm{x}}{dt^2} = -\nabla\Phi.$$

Essentially, this is $\mathrm{F} = m\mathrm{a}$.

Sign Conventions

- East Coast (-+++)
  - Minkowski, Einstein, Pauli, Schwinger
  - Spacelike $ds^2 > 0$
  - Minkowski line element $ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$.
- West Coast (+ - - -)
  - Bjorken-Drell QFT Text - SLAC
  - Timelike $ds^2 > 0$
  - Minkowski line element $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$.