Name: ____ **Score:**

Problem 1. Alice runs with a 20 m pole in the direction of its length. She runs so fast that it appears only 10 m long. She enters the open door of a 10 m barn. Just as the tip of the pole reaches the closed back door, it opens allowing the runner to pass through. From Alice's point of view, the pole is 20 m long and the barn is 5 m long. So, the pole can't fit in the enclosed barn. Explain this paradox quantitatively and using spacetime diagrams.

Problem 2. Let $a^\mu = (-2, 0, 0, 1)$ and $b^\mu = (5, 0, 3, 4)$.

a. Determine if these vectors are timelike, spacelike, or null.

b. Compute $\mathbf{a} \cdot \mathbf{b}$.

Problem 3. A triangle is drawn on a ball of radius 5.0 cm. The sum of the interior angles is $\frac{5\pi}{2}$. What fraction of the surface area does the triangle take up?

Problem 4. The orbital period of a GPS satellite is one-half of a sidereal day or 11 hours 58 minutes. We will assume that Alice, an inertial observer in zero gravitational potential, is watching from a place with no gravitational potential and Bob is on the Earth's surface.

a. Determine the distance from the center of the Earth and the velocity for the satellite to maintain a circular orbit with the given period.

b. If Bob's clock is synchronized with the satellite clock at the beginning of the day, what is the time difference between the clocks at the end of the day?

c. What time difference results from Bob's motion due to the Earth's rotation?

d. Considering the gravitational potentials at the locations of the satellite and Bob, what is the time difference between the initially synchronized clocks at the end of the day?

e. What is the combined effect of these relativistic corrections? What would be the consequence of ignoring these corrections?

Problem 5. A particle moves along the x-axis such that

$$\frac{dx}{dt} = \frac{gt}{\sqrt{1 + g^2 t^2}},$$

for g a constant.

a. Does the particles speed ever exceed c?

b. Calculate the components of the particle's four-velocity.

c. Express x and t as functions of the proper time along the trajectory.

Problem 6. Consider the following coordinate transformation between Cartesian coordinates (x, y) and $(\mu, \nu) : x = \mu\nu$ and $y = \frac{1}{2}(\mu^2 - \nu^2)$.

a. Sketch curves of constant μ and constant ν.

b. Transform the line element $dS^2 = dx^2 + dy^2$ into (μ, ν) coordinates.

c. Are these curves orthogonal?

d. Find the equation of a circle of radius r centered at the origin in these coordinates.

e. Calculate the ratio of the circumference to the diameter of a circle in these coordinates.