PROBLEMS OF SIZE AND SCALE

Why Size is So Important

Table 3.1 Ranges of organism and animal sizes.

	Mass	
Organism	Grams	Units to scale
Mycoplasma	10^{-13}	<0.1 pg
Typical bacterium	10^{-10}	0.1 ng
Tetrahymena	10^{-7}	0.1 μg
Amoeba	10^{-4}	0.1 mg
Rotifer	10^{-4}	0.1 mg
Aphid	10^{-3}	1 mg
Bee	10^{-1}	100 mg
Pygmy shrew	10^{0}	1 g
Hamster	10^{2}	100 g
Human	10^{5}	100 kg
Elephant	5×10^{6}	5000 kg (5 tonnes)
Blue whale	10^{8}	100,000 kg (100 tonnes)

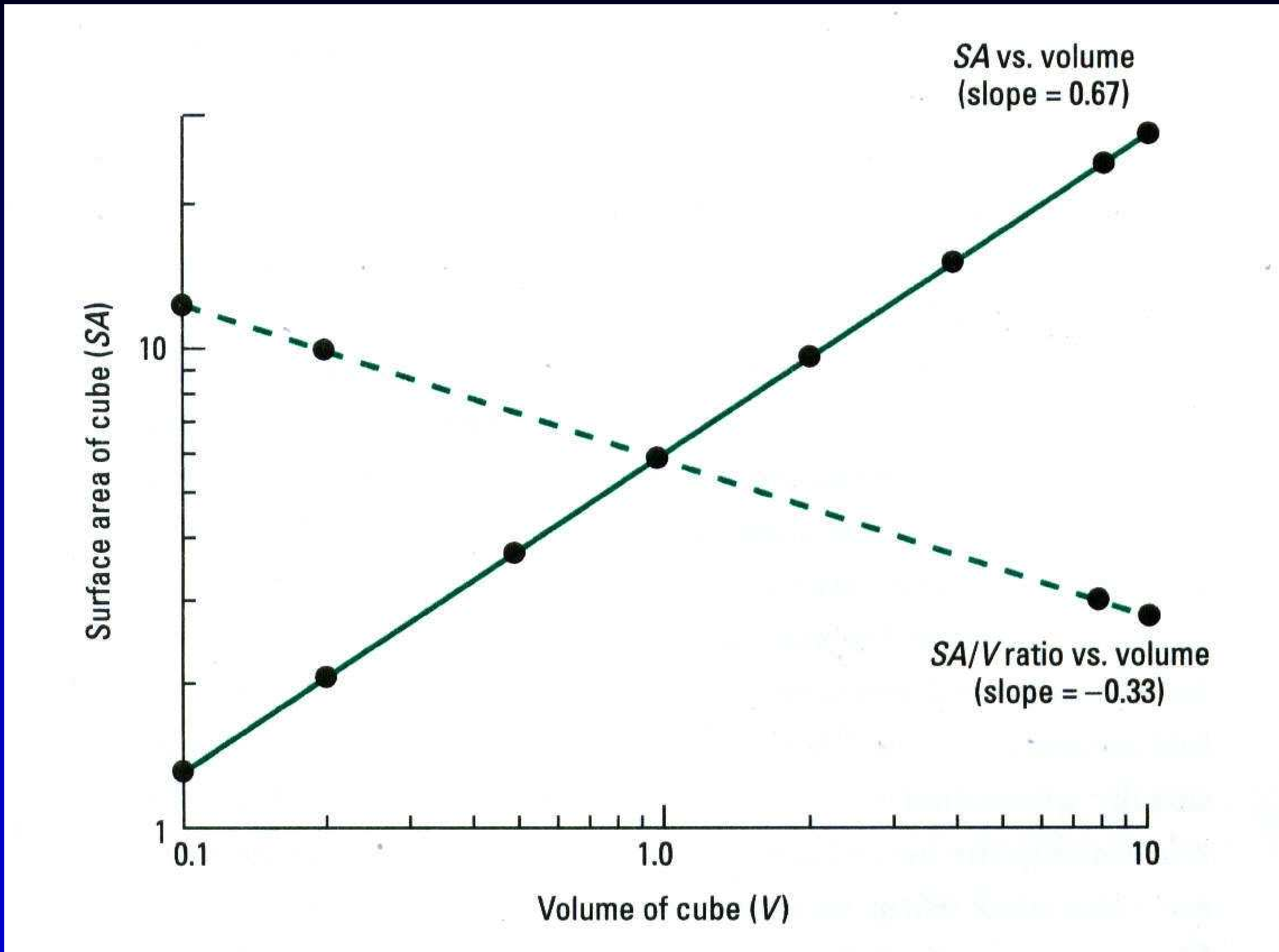
Isometric vs. Allometric Scaling

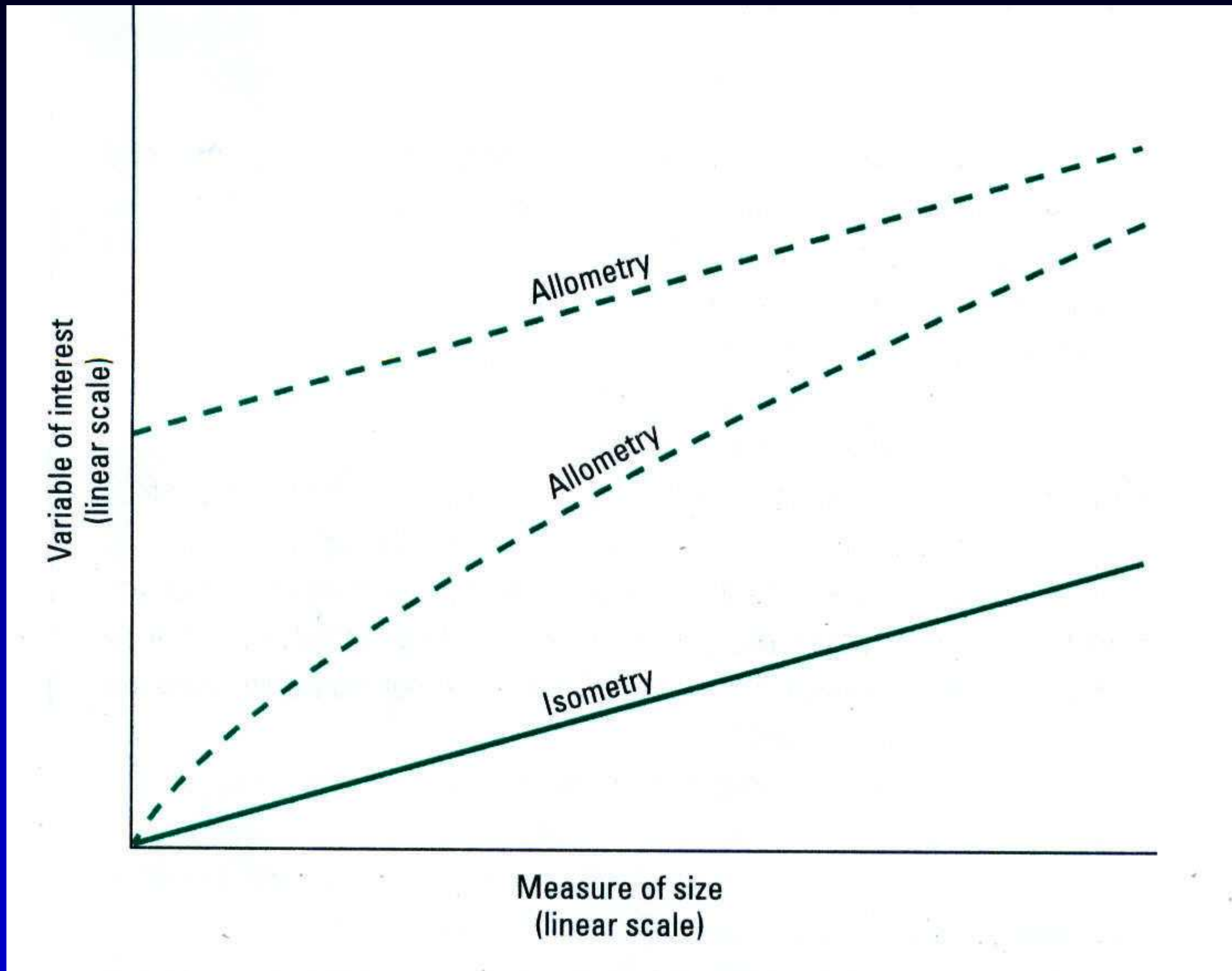
Isometry

- Applies to geometrically similar objects
- Isometric figures always have the same relative proportions (e.g., a cube)

Allometry

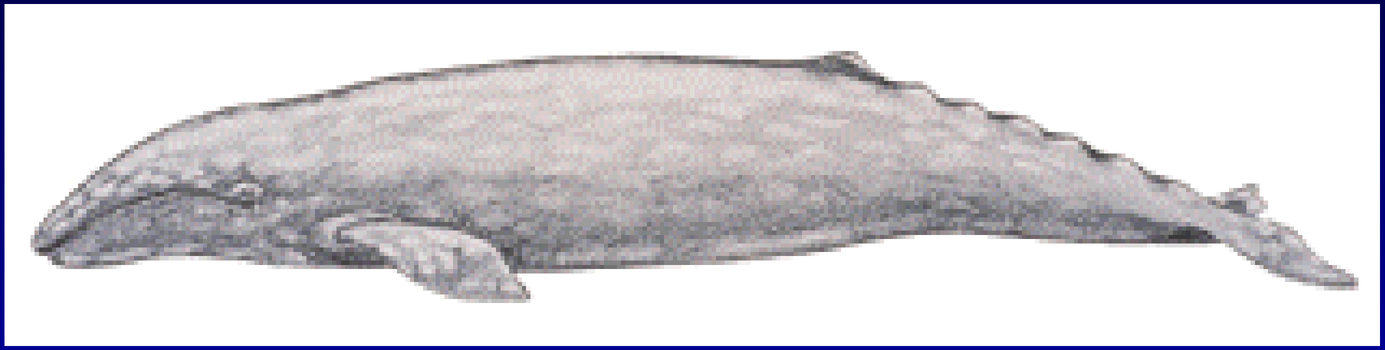
- No simple ratio between two measurements (e.g., gonad size and body mass; metabolism & body mass)
- When plotted, either get straight-line that passes through origin or curvilinear result


SA vs. volume
(slope = 0.67)
SA/V ratio vs. volume
(slope = −0.33)
Surface area of cube (SA)
10
1
0.1
1.0
10
Volume of cube (V)


Allometry
Allometry
Isometry
Variable of interest
(linear scale)
Measure of size
(linear scale)

Metabolic Rate vs. Body Size

Scaling



We know
that a whale

requires more energy (food, calories) per day
than a mouse.

But how much more?

written by Jonathan Swift
in 1726

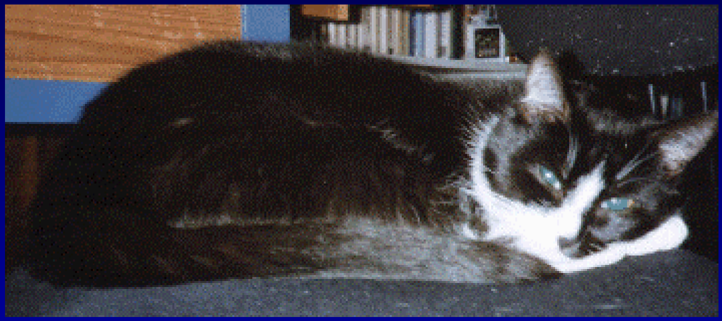
Gulliver in Lilliput -

Lilliputian King is faced
with a physiological problem -

How much to feed Gulliver?

King calculated 1728 Lilliputian portions.

If we’re not careful, we can run afoul
of assumptions - to wit
LSD and the Elephant



The story begins with early research on LSD using cats.

It was discovered that a dose of 0.1 mg of LSD in a 2.6 kg cat caused aggressive behavior.

It was subsequently decided that this might be a good model for studying musth in bull elephants.

Musth is a condition of violent and uncontrollable rage.

The elephant weighed **7722 kg.**

The “investigators” scaled up the dose by **weight**:
(7722 kg /2.6 kg) = 2970
2970 X 0.1 mg = 297 mg
(approximately 1500 X the human dose for a “trip”)

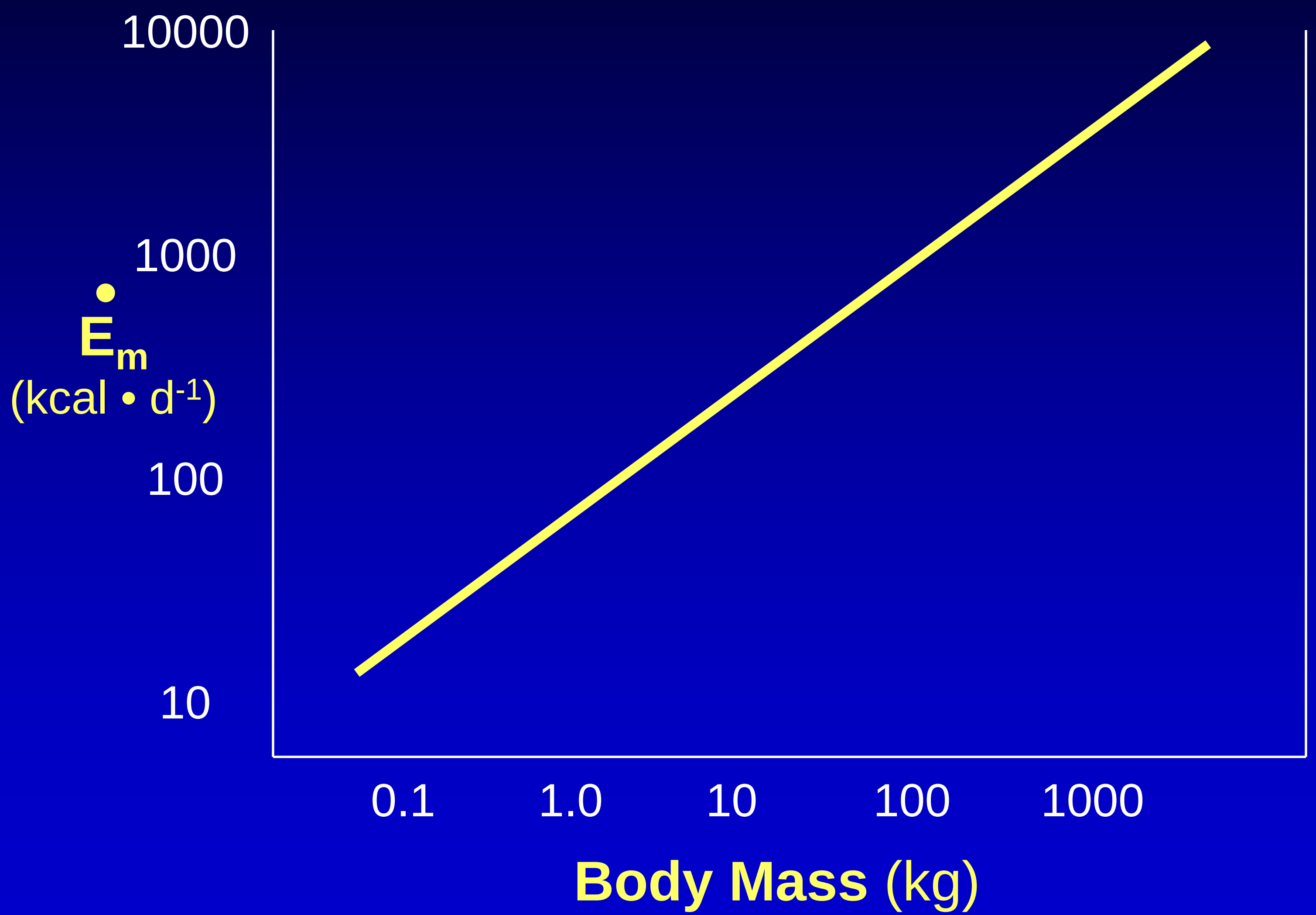
Following injection, the elephant:

- **began trumpeting & running around**
- **stopped**
- **swayed**
- **collapsed**
- **convulsed**
- **defecated, and**
- **died**

The scientists concluded that elephants are particularly sensitive to LSD.

Not so! Assumptions were incorrect!

Metabolic rate (and drug dose) does not scale directly with body weight!



We can see that there is an **exponential** relation between metabolic rate and mass:

$$\dot{V}O_2 \text{ (or } \dot{E}_m) = aM^b$$

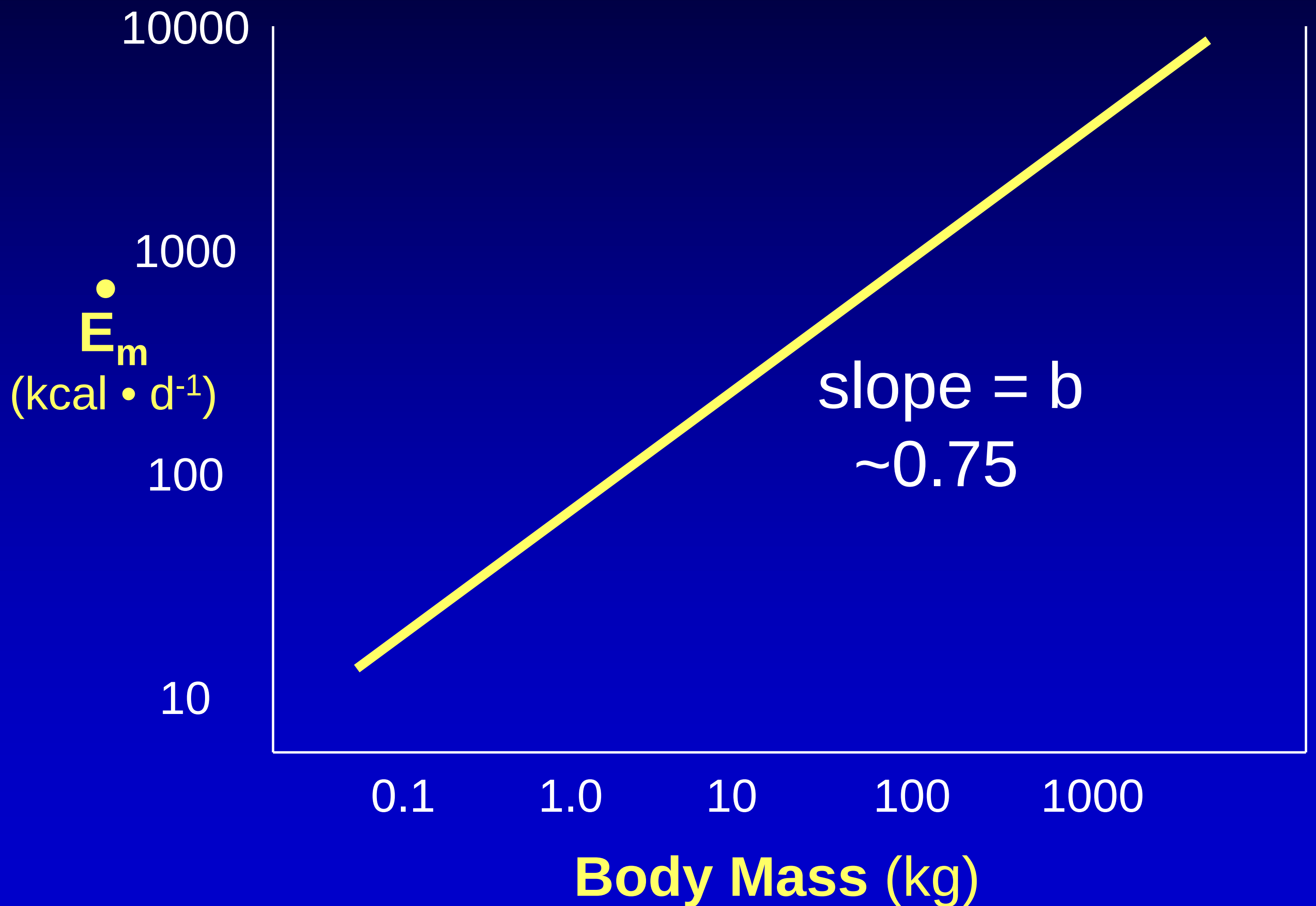
on a log-log plot (such as we just saw) this equation becomes a straight line:

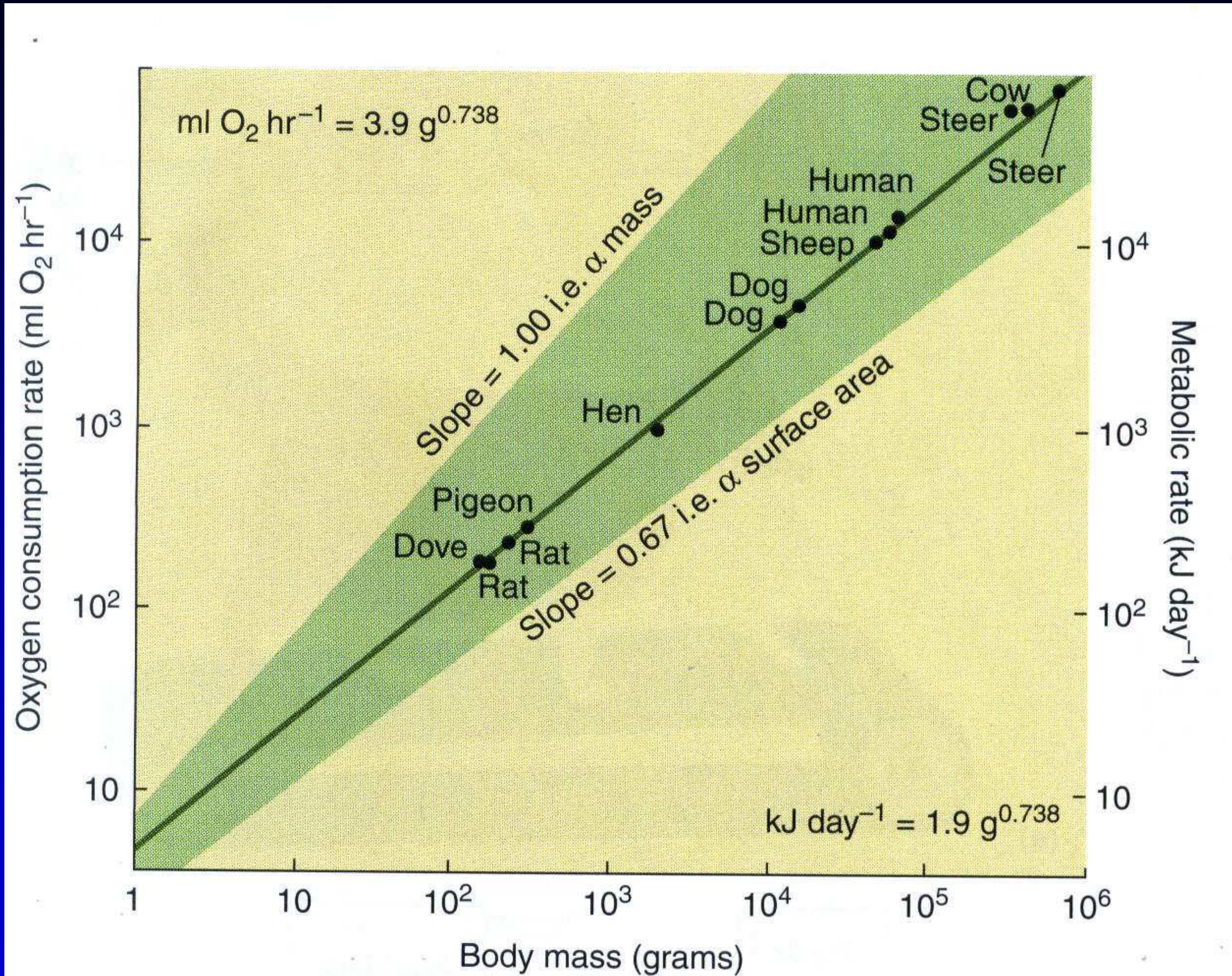
$$\log \dot{V}O_2 = \log(aM^b)$$

$$\log \dot{V}O_2 = \log(a) + b \log(M)$$

which has the form $y = a + bx$, where b is the slope of the line

$$\log \dot{V}O_2 = \log(a) + b \log(M)$$





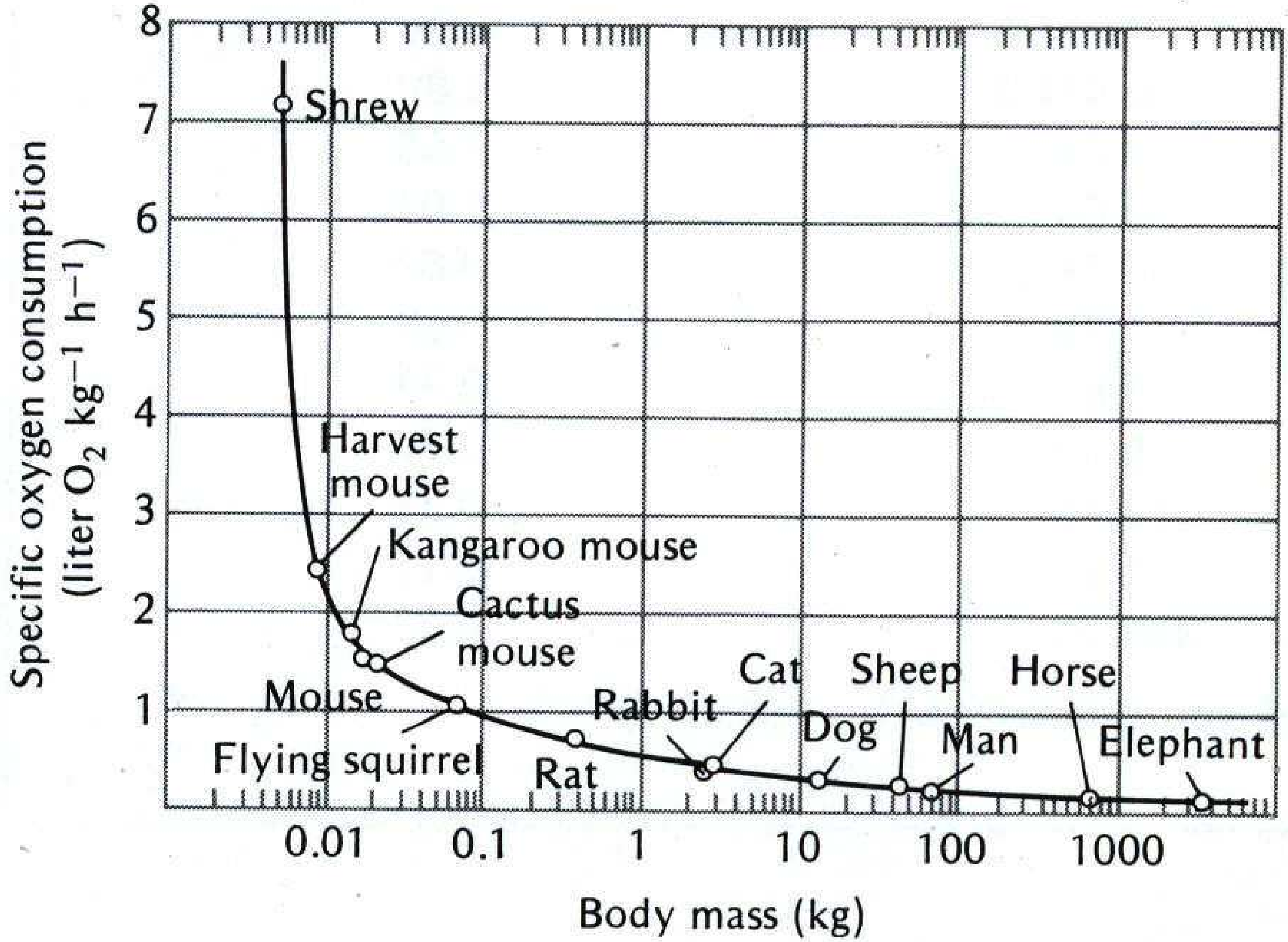
ml O_2 hr^{-1} = 3.9 $g^{0.738}$
Oxygen consumption rate (ml O_2 hr^{-1})
10^4
10^3
10^2
10
Slope = 1.00 i.e. α mass
Slope = 0.67 i.e. α surface area
Cow
Steer
Steer
Human
Human
Sheep
Dog
Dog
Hen
Pigeon
Dove
Rat
Rat
Metabolic rate (kJ day^{-1})
kJ day^{-1} = 1.9 $g^{0.738}$
1
10
10^2
10^3
10^4
10^5
10^6
Body mass (grams)

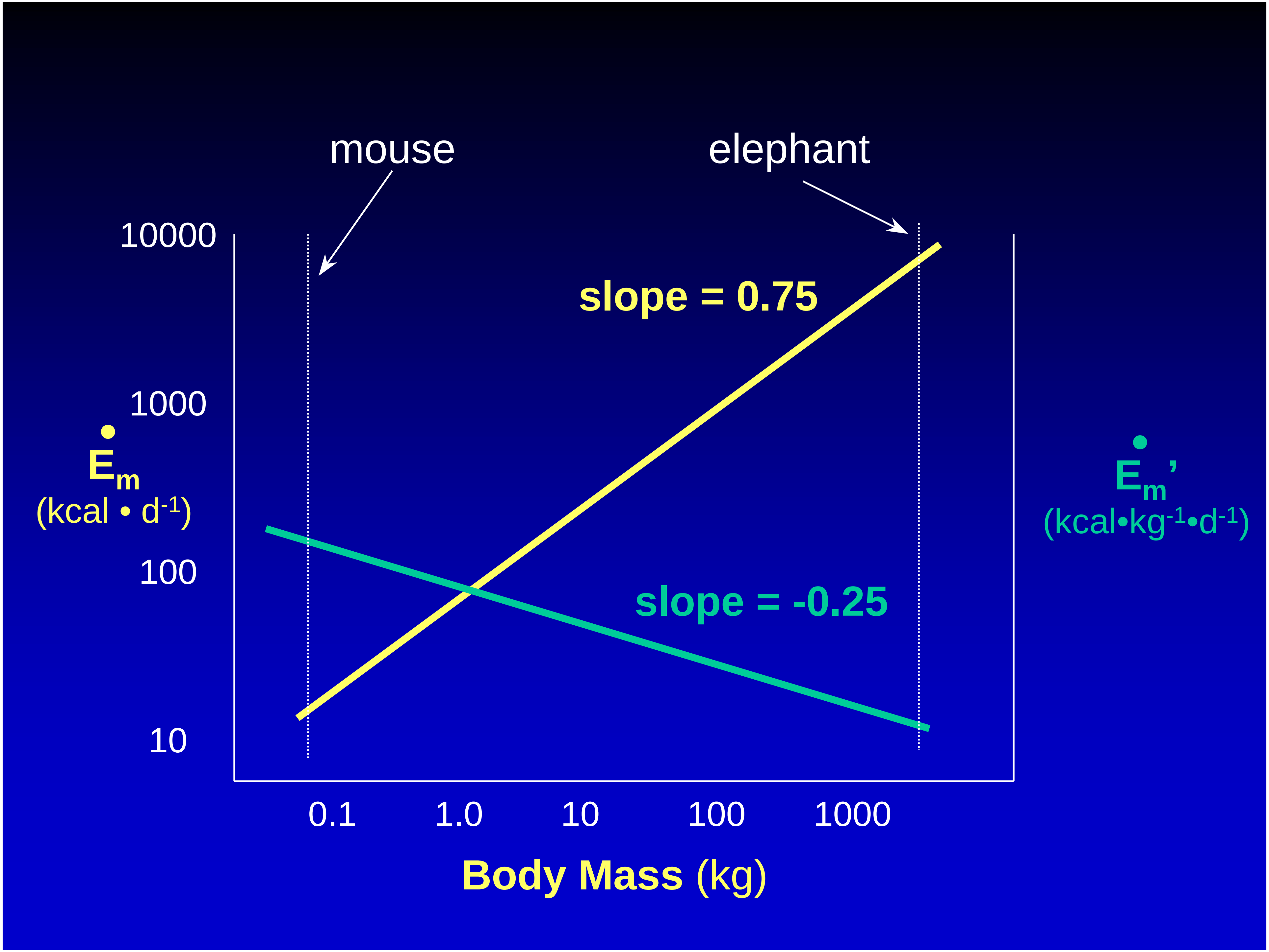
What about the weight-specific metabolic rate?

$$\dot{V}O_2 / M$$

$$(l\ O_2 \bullet h^{-1}) / (kg)$$

$$\dot{V}O_2/M = \frac{aM^{b}}{M} = aM^{b-1} = aM^{-0.25}$$


Specific oxygen consumption
(liter O2 kg−1 h−1)
8
7
6
5
4
3
2
1
Shrew
Harvest mouse
Kangaroo mouse
Cactus mouse
Mouse
Flying squirrel
Rat
Rabbit
Cat
Dog
Sheep
Man
Horse
Elephant
0.01
0.1
1
10
100
1000
Body mass (kg)



mouse
elephant
10000
1000
100
10
Ė_m
(kcal • d-1)
Ė_m'
(kcal•kg-1•d-1)
slope = 0.75
slope = -0.25
0.1
1.0
10
100
1000
Body Mass (kg)

Scaling of Locomotion

- Body size affects relative costs of different kinds of locomotion and their velocity
- Examine running, swimming and flying

Activity vs. $\dot{V}O_2$

Locomotion is expensive:

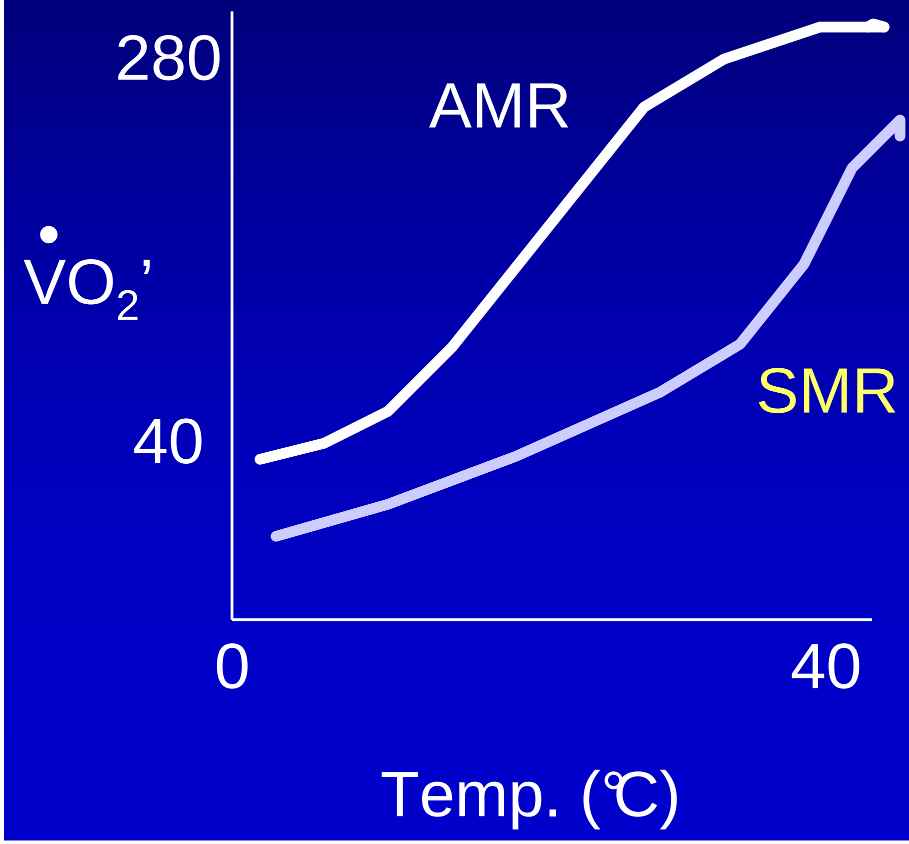
In humans, VO_2 can increase 15-20 X during exercise

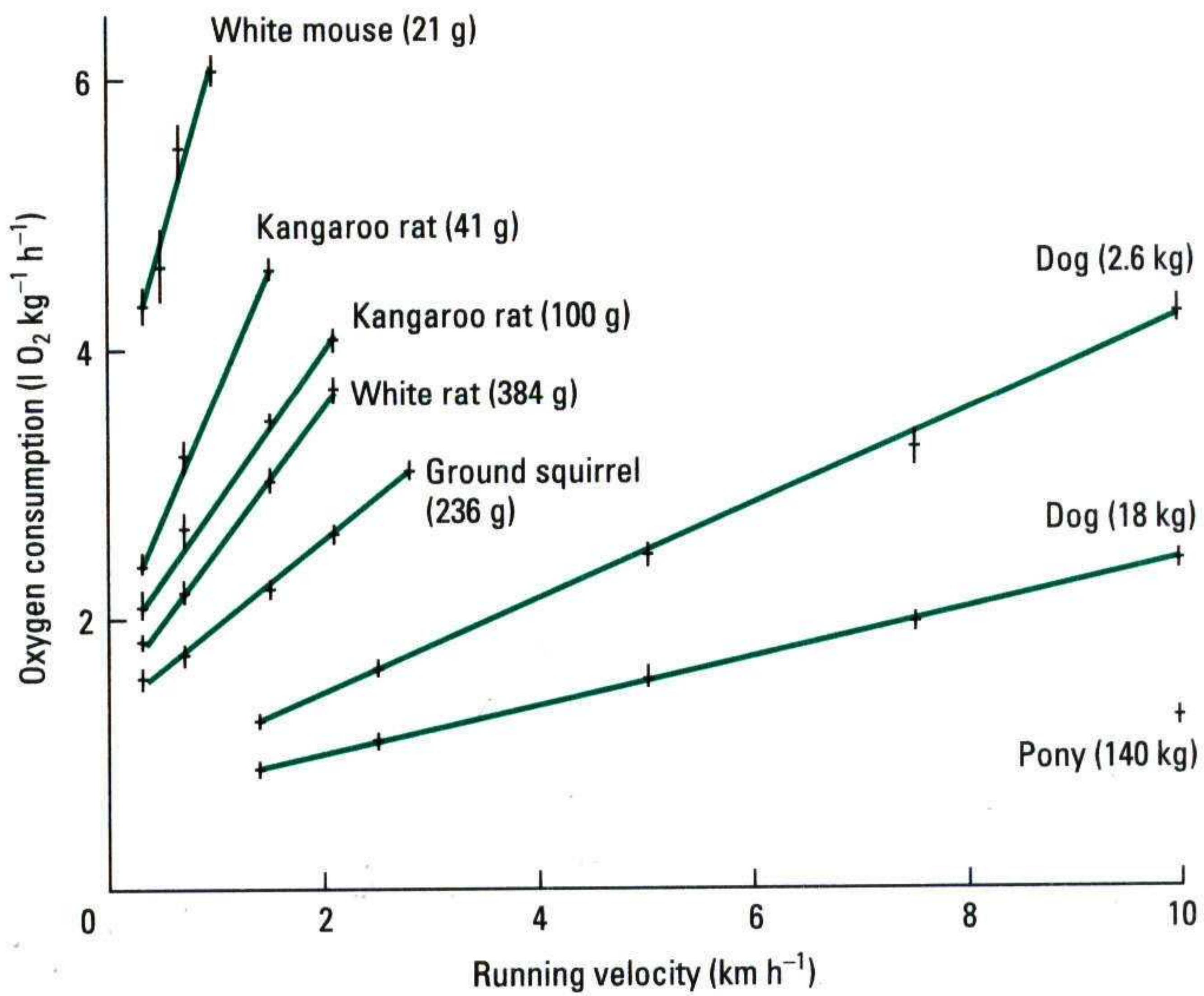
In insects, $\dot{V}O_2$ can increase 50-200 X during flight:

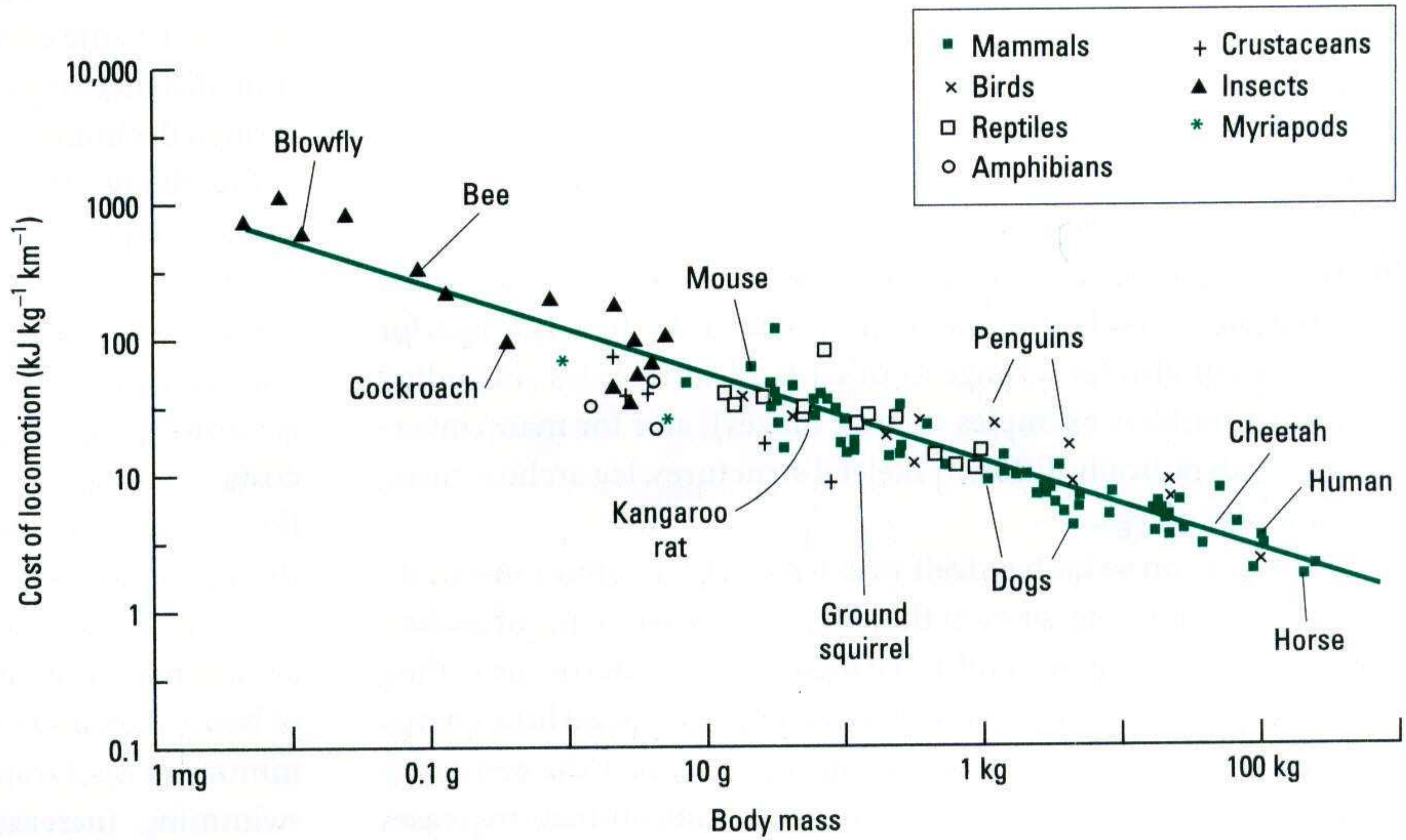
Metabolic Scope for Activity = AMR / SMR

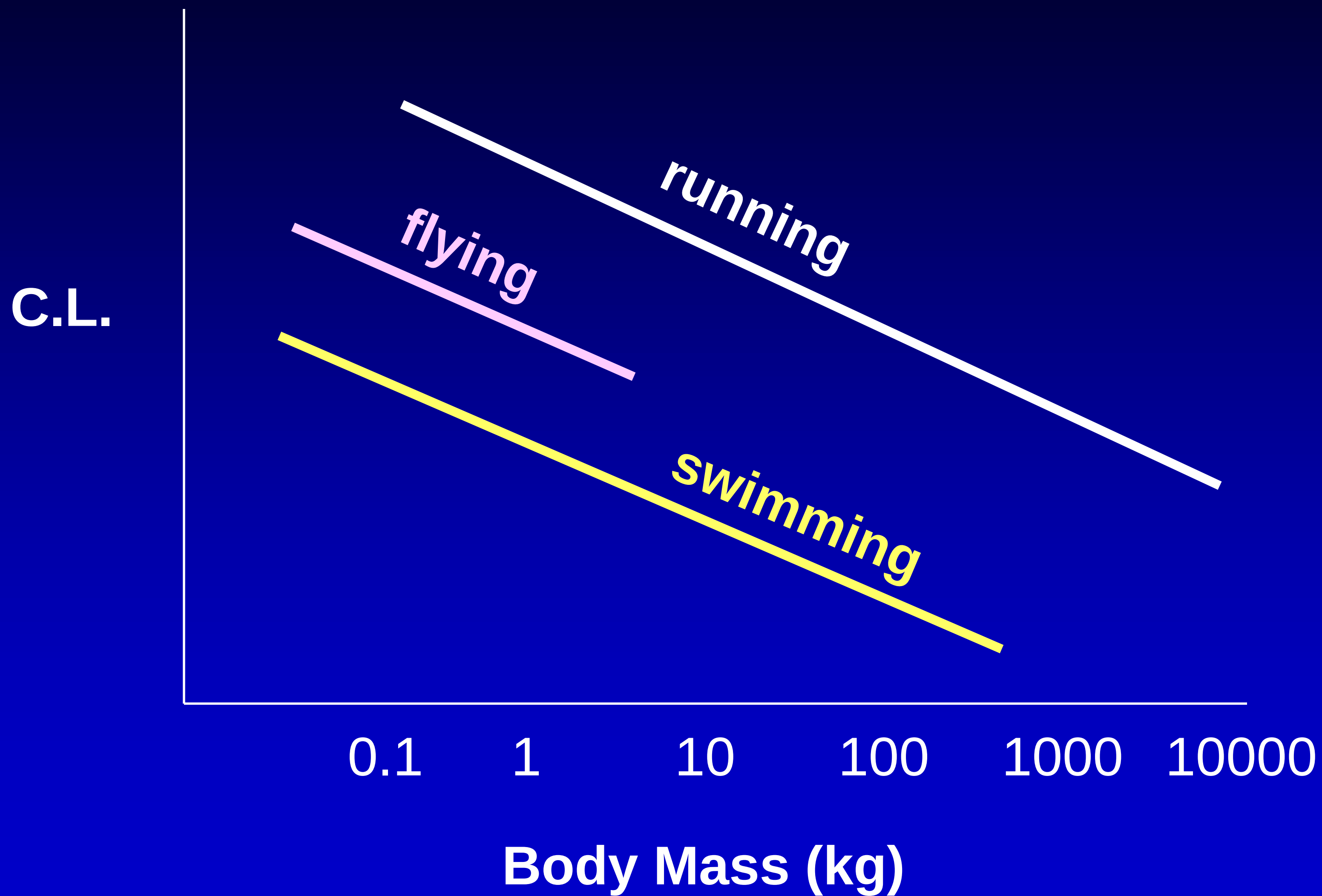
Expressed as dimensionless value (e.g., 8)

Goldfish (ml O_2 • kg^{-1} • h^{-1})




White mouse (21 g)
Kangaroo rat (41 g)
Kangaroo rat (100 g)
White rat (384 g)
Ground squirrel
(236 g)
Dog (2.6 kg)
Dog (18 kg)
Pony (140 kg)
Oxygen consumption (l O_2 kg^{-1} h^{-1})
0
2
4
6
8
10
Running velocity (km h^{-1})


Mammals
Birds
Reptiles
Amphibians
Crustaceans
Insects
Myriapods
10,000
1000
100
10
1
0.1
Cost of locomotion (kJ kg^{-1} km^{-1})
Blowfly
Bee
Cockroach
Mouse
Kangaroo rat
Ground squirrel
Penguins
Dogs
Cheetah
Human
Horse
1 mg
0.1 g
10 g
1 kg
100 kg
Body mass


C.L.
running
flying
swimming
0.1
1
10
100
1000
10000
Body Mass (kg)

Is There An Optimum Size?

Constraints on size

- Basic mechanical design (hydrostatic & exoskeletons vs. endoskeletons)
- Basic physiological design (open vs. closed circ. syst)
- Habitat (aquatic vs. terrestrial)
- Phylogenetic inheritance (insects vs. vertebrates)