MAT 311 Final Exam Spring, 2010

Due 2:30 PM Thursday, April 29

Do five problems from the following six. Show all work for complete credit. This is an open book, open notes exam. You may access to a calculator although no problem requires one.

1. Use the definition of convergence to prove that the sequence

Xn = 

converges.

1. Calculate the limit inferior and limit superior of the sequence

Xn = .

Tell why the sequence does not converge.

1. Tell everything you can about the sequence, ao = 0, a1 = 1, an = (an-1 + an-2)/2 for n ≥ 2.
2. Suppose that f is a bounded function defined on [a,b), that f is bounded below by a positive number r, and that f is Riemann integrable on [a,c] for all c in [a,b). Let bn be a sequence in (a,b) and let zn be the Riemann integral of f on [a,bn]. Prove that if zn is a Cauchy sequence, then bn converges.

Hint: Show | bn– bm |r ≤ | zn– zm | .

1. Let f and g be two differentiable functions on R. Suppose both g and the derivative of g are bounded on R and that the derivative of f is continuous on R. Prove that f(g(x)) is uniformly continuous on the real line. Hint: Use the Mean Value Theorem on the function f(g(x)).
2. Suppose f(x) and g(x) are continuous on [a,b] and f(x) ≤ h(x) ≤g(x) on [a,b].
   1. Prove that the upper and lower Darboux integrals of h(x) exist. Be careful; you cannot assume that h(x) is continuous.
   2. Suppose h(x) = .5 if x is rational and h(x) = .75 is x is irrational. Compute the upper and lower Darboux integrals of h(x) on [0,2].