MAT 162 | Test 3 | Name Key

Summer II 2014

For full credit, show all work.

1. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{\sin(n^4)}{n^2}$

$\sum_{n=1}^{\infty} \left|\frac{\sin(n^4)}{n^2}\right| \le \sum_{n=1}^{\infty} \frac{1}{n^2}$ p = 2 series converges

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$\therefore \sum_{n=1}^{\infty} \frac{\sin(n^4)}{n^2}$ converges absolutely

reason 6

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(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n^4+1}{\sqrt{n^9}+6}$

$f(x) = \frac{x^4+1}{x^{9/2}+6} > 0$; cont. & $f'(x) = \frac{(x^{9/2}+6)4x^3 - (x^4+1)\frac{9}{2}x^{7/2}}{(x^{9/2}+6)^2}$

$= \frac{-\frac{1}{2}x^{15/2} + 24x^3 - \frac{9}{2}x^{9/2}}{(x^{9/2}+6)^2} < 0$

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converges by Alt. Series

but not abs convergent

$\sum \frac{n^4}{2n^{9/2}} < \sum \frac{n^4+1}{\sqrt{n^9}+6}$ $\therefore$ diverges by comparison Test

$= \frac{1}{2}\sum \frac{1}{n^{1/2}}$

↑ diverges

$\therefore$ conditionally convergent

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(c) $\sum_{n=1}^{\infty} \left(\frac{-n}{2n+3}\right)^n$

$\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \lim_{n\to+\infty} \left|\frac{-n}{2n+3}\right| = \frac{1}{2}$

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converges absolutely by n-th root test.

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2. Find the radius and interval of convergence for $f(x) = \sum_{n=1}^{\infty} (3x-4)^n 2^{-n}(1/n)$.

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{|3x-4|^{n+1}}{|3x-4|^n}\frac{2^{-(n+1)}}{2^{-n}}\frac{1}{n+1}n = \frac{|3x-4|}{2}\frac{n}{n+1}$ (3)

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$\lim\left|\frac{a_{n+1}}{a_n}\right| = \frac{|3x-4|}{2} < 1$ (3)

$|3x-4| < 2$

$|x - \frac{4}{3}| < \frac{2}{3}$

$-\frac{2}{3} < x - \frac{4}{3} < \frac{2}{3}$

$\frac{4}{3} - \frac{2}{3} < x < \frac{4}{3} + \frac{2}{3}$

$\frac{2}{3} < x < 2$ (2)

(1) at $x = 2$, diverge $\sum \frac{1}{n}$

(1) at $x = \frac{2}{3}$ converge $\sum (-1)^n \frac{1}{n}$

$[\frac{2}{3}, 2)$ (1)

$R = \frac{2}{3}$ (1)

3. Use a power series to estimate $\int_0^{0.1} \frac{\sin(x)}{x}\, dx$ with an error less than 10^{-5}.

$\sin x = x - \frac{x^3}{3!} - - - = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$ (3)

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$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} - - -$

$= \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k+1)!}$ (3)

$\int_0^{0.1} \frac{\sin x}{x} dx = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!\, 2k+1}\Big|_0^{.1} = \sum_{k=0}^{\infty} (-1)^k \frac{(.1)^{2k+1}}{(2k+1)!\, 2k+1}$ (2)

$= .1 - \frac{(.1)^3}{3!(3)} + \frac{(.1)^5}{5!(5)} + - -$ (2) ↖ $\leq 10^{-5}$

Answer is $.1 - \frac{(.1)^3}{18}$ (2)

4. Use the fact that the following series is a telescoping series to calculate $\sum_{n=3}^{\infty} \frac{1}{n^2+2n}$ exactly.

$\frac{1}{n^2+2n} = \frac{A}{n} + \frac{B}{n+2} = \frac{1}{2}\left[\frac{1}{n} - \frac{1}{n+2}\right]$ (3)

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$1 = A(n+2) + Bn$

$n = 0 \quad 1 = A(2) \Rightarrow A = \frac{1}{2}$

$n = -2 \quad 1 = B(-2) \Rightarrow B = -\frac{1}{2}$

$S_5 = \frac{1}{2}[\frac{1}{3} - \frac{1}{5}] + \frac{1}{2}[\frac{1}{4} - \frac{1}{6}] + \frac{1}{2}[\frac{1}{5} - \frac{1}{7}] + \frac{1}{2}[\frac{1}{6} - \frac{1}{8}] + \frac{1}{2}[\frac{1}{7} - \frac{1}{9}]$ (3)

$S_n = \frac{1}{2}[\frac{1}{3} + \frac{1}{4}] + \frac{1}{2}[-\frac{1}{n+1} - \frac{1}{n+2}]$ (3)

$\lim S_n = \frac{1}{2}[\frac{1}{3} + \frac{1}{4}]$

Answer $\frac{1}{2}[\,] = \frac{7}{24}$ (3)

$\frac{7}{12}$

5. Calculate $\sum_{n=1}^{\infty} \frac{2^{n-2}7^{n+4}}{3^{5n}}$ exactly.

$2^{-2}7^4 \sum_{n=1}^{\infty} \frac{2^n \cdot 7^n}{(3^5)^n}$

(4)

$2^{-2}7^4 \frac{\frac{(2)(7)}{3^5}}{1-\frac{(2)(7)}{3^5}} = \frac{2^{-1}7^5}{3^5-(2)(7)} = \frac{7^5}{8(3^5-14)} = \frac{16807}{1832} = 9.32486 \approx 37.307$

6. Use the integral test to determine the number of terms in the partial sum for $\sum_{n=1}^{\infty} \frac{1}{n^6}$ that will estimate the infinite series with an error less than .005

$\int_n^{+\infty} \frac{1}{x^6}\,dx = \lim_{b\to+\infty} \left.\frac{-x^{-5}}{5}\right|_n^b$

(14)

$= \lim_{b\to+\infty} \frac{-b^{-5}}{5} + \frac{n^{-5}}{5}$

$= \frac{1}{5}\frac{1}{n^5} < .005$

$40 < n^5$

$40^{\frac{1}{5}} < n$

$2.09 < n$

$3 \leq n$

$1 + \frac{1}{2^6} + \frac{1}{3^6}$