Test 2 Smmer II, 2014 Name: Key

MAT 162 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. The test is closed book.

1. Use Euler's Method to approximate y(1.2) given $\frac{dy}{dx} = y^2 - 5x$, and y(1) = 2. Use a stepsize of 0.1 .

x	y	f(x, y) · .1
1	2	$(2^2-5)(.1) = -.1$
1.1	1.9	$[(1.9)^2 - 5.5] \cdot .1 = -.189$
1.2	1.711	

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2. Find y(x), the solution to $\frac{dy}{dx} = (\cos y)^2(x^2 + 1)$ ' y(0) = π/4 .

3 $\int \sec^2 y \, dy = \int (x^2+1)\,dx$ 3

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12 2 $\tan y = \frac{1}{3}x^3 + x + c$

$1 = c$

$\tan y = \frac{1}{3}x^3 + x + 1$

$\boxed{y = \arctan\left(\frac{1}{3}x^3 + x + 1\right)}$

3. Find y(x), the solution to $\frac{dy}{dx} = 2xe^x + y$ ' y(1) = 0 .

$\frac{dy}{dx} - y = 2xe^x \quad , y(1) = 0$

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$\frac{d}{dx}(e^{-x}y) = 2x$

$e^{-x}y = x^2 + c$

$e^{-1}(0) = 1^2 + c$

$c = -1$

$e^{-x}y = x^2 - 1$

$\boxed{y = e^x(x^2 - 1)}$

4. A tank is filled with 200 liters of contaminated water containing 3 g of toxins. Water containing .004 g of toxin per liter is pumped in at a rate of 10 l/min., mixes instantaneously, and then is pumped out at the same rate. Find y(t) the number of grams of the toxin in the tank t minutes after the rinse begins. Then find the time at which there is 2 g of toxin present. When will there be ½ g of toxin present?

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$S(0) = 3$

$\frac{dS}{dt} = .04 - .05S$

$\frac{dS}{dt} + .05S = .04$

$I(t) = e^{.05t}$

$\frac{d}{dt}(e^{.05t}S) = .04e^{.05t}$

$e^{.05t}S = .8e^{.05t} + C$

$S = .8 + Ce^{-.05t}$

$3 = S(0) = .8 + Ce^{0}$

$2.2 = C$

$\boxed{S(t) = .8 + 2.2e^{-.05t}}$ 2

$2 = .8 + 2.2e^{-.05t}$

$1.2 = 2.2e^{-.05t}$

$\frac{12}{22} = e^{-.05t}$

$\ln\frac{12}{22} = -.05t$

$\boxed{-20\ln\frac{12}{22} = t}$ 2

$t = 12.12$ min.

2 $S(t) > .8$

$\therefore$ There will never be $\frac{1}{2}$ g present.

5. First find the solution to $2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 2y = 0$, $y(0) = 0$, $y'(0) = 1$.

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$2r^2 - 3r - 2 = 0$

$(2r+1)(r-2) = 0$

3 $r = -\frac{1}{2}, \quad r = 2$

3 $y(x) = c_1e^{-\frac{1}{2}x} + c_2e^{2x}$

$y'(x) = -\frac{1}{2}c_1e^{-\frac{1}{2}x} + 2c_2e^{2x}$

$0 = y(0) = c_1 + c_2$

$1 = -\frac{1}{2}c_1 + 2c_2$

2 $\begin{cases} 0 = c_1 + c_2 \\ 1 = -\frac{1}{2}c_1 + 2c_2 \end{cases} \Rightarrow$

$0 = c_1 + c_2$

$2 = -c_1 + 4c_2$

$2 = 5c_2$

$c_2 = \frac{2}{5}$

2 $c_1 = -\frac{2}{5}$

$\boxed{y(x) = -\frac{2}{5}e^{-\frac{1}{2}x} + \frac{2}{5}e^{2x}}$

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6. Find the value of k so that $f(x) = x^{-8} + kx^{-9}$ is a probability density function on $[1,+\infty)$ and then find the value of the mean for the probability density function.

$$1 = \int_1^{+\infty} x^{-8} + kx^{-9}\,dx = \lim_{b\to+\infty} \frac{x^{-7}}{-7} + \frac{k}{-8}x^{-8}\Big|_1^b = \frac{1}{7} + \frac{k}{8}$$

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$$\boxed{\frac{6}{7} = \frac{k}{8} \qquad k = \frac{48}{7}}$$

$$\mu = \int_1^{+\infty} x\left(x^{-8} + \frac{48}{7}x^{-9}\right)dx$$

$$= \lim_{b\to+\infty}\left(\frac{x^{-6}}{-6} + \frac{48}{7}\frac{x^{-7}}{-7}\right)\Big|_1^b = \frac{1}{6} + \frac{48}{49} = \boxed{\cancel{\frac{48}{42}}} \rightarrow \frac{49 + 6(48)}{6(49)}$$

$$= \frac{337}{294} = 1.146$$

7. Find the area of the surface generated by rotating about the x-axis the graph of $y = 1+8x$ from 0 to $\pi/4$.

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$$\int_0^{\frac{\pi}{4}} 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

$$\int_0^{\frac{\pi}{4}} 2\pi(1+8x)\sqrt{1+8^2}\,dx$$

$$2\pi\sqrt{65}\,(x+4x^2)\Big|_0^{\frac{\pi}{4}} = 2\pi\sqrt{65}\left(\frac{\pi}{4}+4\left(\frac{\pi}{4}\right)^2\right)$$

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$$= 2\pi^2\sqrt{65}\,\frac{1}{4}(1+\pi) = \frac{\pi^2}{2}\sqrt{65}(1+\pi)$$

8. Let A be the triangle in the x-y plane whose corners are (0,0), (-2,5), and (1,1). Suppose A has a uniform mass density ρ. Find the moment about the x-axis and the moment about the y-axis. You need only set up the integrals; you do not have to evaluate them.

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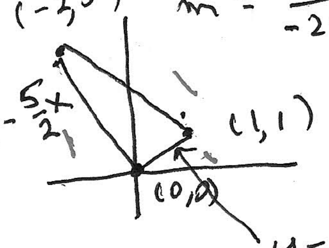

$$m = \frac{5-1}{-2-1} = \frac{4}{-3} = -\frac{4}{3}$$

$$y - 1 = -\frac{4}{3}(x-1)$$

$$y = -\frac{4}{3}x + \frac{4}{3} + 1$$

$$= -\frac{4}{3}x + \frac{7}{3}$$

$$4\ M_y = \int_{-2}^{0} \rho x\left(-\frac{4}{3}x + \frac{7}{3} - \left(-\frac{5}{2}x\right)\right)dx + \int_0^1 \rho x\left(-\frac{4}{3}x + \frac{7}{3} - x\right)dx$$

$$4\ M_x = \int_{-2}^{0} \frac{\rho}{2}\left(\left(-\frac{4}{3}x+\frac{7}{3}\right)^2 - \left(-\frac{5}{2}x\right)^2\right)dx + \int_0^1 \frac{\rho}{2}\left(\left(-\frac{4}{3}x+\frac{7}{3}\right)^2 - x^2\right)dx$$