

Directions: Show all work for partial credit purposes. You may use a graphing calculator and notes recorded on one side of a single 8.5 by 11 inch paper. Otherwise the test is closed book. When you turn in your test, staple your notes to this sheet.

For 1-4, calculate the following:

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1. $\int x \sin(15x) dx$

$$= uv - \int v du = -\frac{1}{15} x \cos(15x) - \int -\frac{1}{15} \cos(15x) dx$$

$$= -\frac{1}{15} x \cos(15x) + \frac{1}{15^2} \sin(15x) + C$$

$u = x \quad dv = \sin(15x) dx$
 $du = dx \quad \frac{dv}{dx} = \sin(15x)$
 $v = -\frac{1}{15} \cos(15x)$

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2. $\int \tan(x) \sec^4(x) dx$

$$= \int \tan(x) (\sec^2(x)) \sec^2(x) dx \quad \text{or} \quad \int \sec^3(x) \tan(x) \sec^2(x) dx$$

$$= \int \tan(x) (\tan^2(x) + 1) \sec^2(x) dx$$

$u = \tan(x) \quad du = \sec^2(x) dx$
 $= \int u(u^2 + 1) du = \frac{1}{4} u^4 + \frac{1}{2} u^2 + C = \frac{1}{4} \tan^4(x) + \frac{1}{2} \tan^2(x) + C$

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3. $\int x^3 \sqrt{25-x^2} dx$

$$= \int 5^3 \sin^3 \theta \cdot 5 \cos \theta \cdot 5 \cos \theta d\theta$$

$$= 5^5 \int \sin^2 \theta \cos^2 \theta \sin \theta d\theta$$

$$= 5^5 \int (1 - \cos^2 \theta) \cos^2 \theta \sin \theta d\theta$$

$$= 5^5 \int (1 - u^2) u^2 (-du)$$


$$= 5^5 \int (u^4 - u^2) du$$

$$= 5^5 \left(\frac{1}{5} u^5 - \frac{1}{3} u^3 \right) + C$$

$$= 5^5 \left(\frac{1}{5} \cos^5 \theta - \frac{1}{3} \cos^3 \theta \right) + C$$

$$= 5^5 \left(\frac{1}{5} \left(\frac{\sqrt{25-x^2}}{5} \right)^5 - \frac{1}{3} \left(\frac{\sqrt{25-x^2}}{5} \right)^3 \right) + C$$

$x = 5 \sin \theta$
 $\sqrt{25-x^2} = 5 \cos \theta$
 $dx = 5 \cos \theta d\theta$
 $u = \cos \theta$
 $du = -\sin \theta d\theta$



4. $\int \frac{x^2}{x^2+17x+30} dx = \int \frac{x^2+17x+30 - 17x-30}{x^2+17x+30} dx$

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$= \int \left(1 - \frac{17x+30}{(x+15)(x+2)} \right) dx = \int 1 - \left(\frac{75}{13} \frac{1}{x+15} + \frac{16}{13} \frac{1}{x+2} \right) dx$

$\frac{7x+30}{(x+15)(x+2)} = \frac{A}{x+15} + \frac{B}{x+2}$
 $7x+30 = A(x+2) + B(x+15)$
 $x=-15 \implies -105+30 = A(-13) \implies A = \frac{16}{13}$
 $x=-2 \implies 16 = B(13) \implies B = \frac{75}{13}$

5. Estimate $\int_1^3 e^{\sin(x)} dx$ using the Simpsons Rule with $n=6$. Write the sum; you do not

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have to evaluate the sum.

$\Delta x = \frac{3-1}{6} = \frac{1}{3}$

$\frac{1}{3} (f(1) + 4f(\frac{4}{3}) + 2f(\frac{5}{3}) + 4f(2) + 2f(\frac{7}{3}) + f(3))$ when $f(x) = e^{\sin(x)}$

6. Calculate the following; if the integral does not converge, state "does not converge."

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a. $\int_1^{+\infty} x^3 e^{-x^4} dx = \lim_{b \rightarrow +\infty} \int_1^b x^3 e^{-x^4} dx = \lim_{b \rightarrow +\infty} -\frac{1}{4} e^{-x^4} \Big|_1^b = \frac{1}{4}$

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converges

b. $\int_3^4 \frac{4}{\sqrt{x-3}} dx = \lim_{a \rightarrow 3^+} \int_a^4 \frac{4}{\sqrt{x-3}} dx = \lim_{a \rightarrow 3^+} 8\sqrt{x-3} \Big|_a^4 = 8$

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7. Tell why the following converge or diverge:

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a. $\int_1^{+\infty} \frac{x^2+1}{(x^2+24)^2} dx$ compare to $\frac{1}{x^2}$

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b. $\int_1^{+\infty} \frac{x^2+1}{(x+24)^3} dx$ compare to $\frac{1}{x}$

compare to $\frac{1}{x}$

$\frac{x^2+1}{(x^2+24)^2} \leq \frac{x^2+x^2}{(x^2)^2} = \frac{2}{x^2}$ $p=2$
 $\int_1^{+\infty} \frac{2}{x^2} dx$ converges

$\frac{1}{25x} = \frac{1}{25} \frac{1}{x}$ $p=1$

$\therefore \int \frac{x^2+1}{(x^2+24)^2} dx$ converges

$\therefore \int \frac{x^2+1}{(x+24)^3} dx$ diverges

$\leq \frac{x^2+1}{(x+24)^2}$

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8. Calculate $\int \frac{x+3}{x^2+4x+8} dx$

$$u = x+2$$
~~$$u = x+2$$~~

$$du = dx$$

$$= \int \frac{x+3}{(x+2)^2+2^2} = \int \frac{x+2+1}{(x+2)^2+2^2} du$$

$$= \frac{1}{2} \int \frac{2u}{u^2+2^2} du + \int \frac{1}{u^2+2^2} du$$

$$= \frac{1}{2} \ln(u^2+2^2) + \frac{1}{2} \arctan \frac{u}{2}$$

$$= \frac{1}{2} \ln(x^2+4x+8) + \frac{1}{2} \arctan \left(\frac{x+2}{2} \right) + C$$

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9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the

integral): $\int \frac{4x+5}{(x^2+12x-28)^3(x^2+2x+10)^2} dx$

$$\frac{4x+5}{(x+14)^3(x-2)^3((x+1)^2+3^2)^2}$$

$$= \frac{A}{x+14} + \frac{B}{(x+14)^2} + \frac{C}{(x+14)^3}$$

$$+ \frac{Ex+F}{x^2+2x+10} + \frac{Gx+H}{(x^2+2x+10)^2}$$