Due Thursday 9AM

$$\int (5x^2 + 2x + 1)\sin(3x)\,dx$$

$$\int \sin^6 x \cos^3 x\,dx$$

$$\int \tan^3 x \sec^2 x\,dx$$

$$\int \frac{1}{x^2\sqrt{x^2+36}}\,dx$$

$$\int \frac{\sqrt{25-x^2}}{x^3}\,dx$$

$$\int \frac{3x+2}{x^2+7x+10}\,dx$$

(I) $\int (5x^2+2x+1)\sin(3x)\,dx \quad = uv - \int v\,du$

$u = 5x^2+2x+1 \qquad dv = \sin(3x)dx \qquad = (5x^2+2x+1)\left[\frac{\cos(3x)}{3}\right]$

$\frac{du}{dx} = 10x+2 \qquad \frac{dv}{dx} = \sin(3x)$

$du = (10x+2)dx \qquad v = -\frac{\cos(3x)}{3} \qquad - \int \left(-\frac{\cos(3x)}{3}\right)(10x+2)dx$

$* \therefore \int (5x^2+2x+1)\sin(3x)\,dx = \frac{(5x^2+2x+1)}{3}(-\cos(3x)) + \frac{1}{3}\int (10x+2)\cos(3x)\,dx$

$\int (10x+2)\cos(3x)\,dx \quad = \frac{1}{3}(10x+2)(\sin 3x) - \frac{1}{3}\int \sin(3x)\,10\,dx$

$u = 10x+2 \qquad dv = \cos(3x)\,dx$

$\frac{du}{dx} = 10 \qquad \frac{dv}{dx} = \cos(3x) \qquad = \frac{1}{3}(10x+2)\sin(3x) + \frac{10}{9}\cos(3x) +$

$du = 10\,dx \qquad V = \frac{1}{3}\sin(3x)$

$\therefore \int = \left(\frac{5x^2+2x+1}{3}\right)(-\cos(3x)) + \frac{1}{3}\left[\frac{1}{3}(10x+2)\sin(3x) + \frac{10}{9}\cos(3x)\right] + C$

or

$= -\frac{\cos(3x)(5x^2+2x+1)}{3} + \frac{1}{9}(10x+2)\sin(3x) + \frac{10}{27}\cos(3x) + C$

(II) $\int \sin^6 x \cos^3 x\,dx = \int \sin^6 x\,(1-\sin^2 x)\cos x\,dx$

$u = \sin x \qquad = \int u^6(1-u^2)\,du$

$\frac{du}{dx} = \cos x \qquad = \int (u^6 - u^8)\,du = \frac{1}{7}u^7 - \frac{1}{9}u^9 + C$

$du = \cos x\,dx \qquad = \boxed{\frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C}$

(III) $\int \tan^3 x \sec^2 x\,dx = \int u^3\,du$

$u = \tan x \qquad = \frac{1}{4}u^4 + C$

$\frac{du}{dx} = \sec^2 x \qquad = \boxed{\frac{1}{4}\tan^4 x + C}$

$du = \sec^2 x\,dx$

IV $\displaystyle\int \frac{1}{x^2\sqrt{x^2+36}}\,dx$

$x = 6\tan\theta$

$x^2+36 = 36(\tan^2\theta+1) = 36\sec^2\theta$

$\sqrt{x^2+36} = 6\sec\theta$

$dx = 6\sec^2\theta\,d\theta$

$$= \int \frac{1}{6^2\tan^2\theta\,6\sec\theta}\,6\sec^2\theta\,d\theta$$

$$= \int \frac{1}{6^2}\,\frac{1}{\tan^2\theta}\,\sec\theta\,d\theta$$

$$= \frac{1}{6^2}\int \frac{\cos^2\theta}{\sin^2\theta}\,\frac{1}{\cos\theta}\,d\theta$$

$$= \frac{1}{36}\int \frac{1}{\sin^2\theta}\cos\theta\,d\theta$$

$w = \sin\theta$

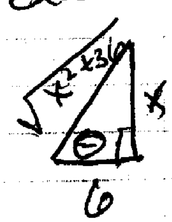
$\frac{dw}{d\theta} = \cos\theta$

$dw = \cos\theta\,d\theta$

$$= \frac{1}{36}\int w^{-2}\,dw$$

$$= \frac{1}{36}\left(-\frac{1}{w}\right) + C$$

$$= -\frac{1}{36\sin\theta} + C$$



$\tan\theta = \frac{x}{6}$

$$= \boxed{-\frac{1}{36}\,\frac{\sqrt{x^2+36}}{x} + C}$$

(V) $\int \frac{\sqrt{25-x^2}}{x^3}\,dx = \int \frac{5\cos\theta \; 5\cos\theta}{5^3 \sin^3\theta} = \frac{1}{5}\int \frac{\cos^2\theta \sin\theta}{\sin^4\theta}$

$x = 5\sin\theta$

$\sqrt{25-x^2} = \sqrt{25-25\sin^2\theta} = 5\cos\theta$

$dx = 5\cos\theta\, d\theta$

$$= \frac{1}{5}\int \frac{\cos^2\theta}{(1-\cos^2\theta)^2}\sin\theta\, d\theta$$

$w = \cos\theta$

$dw = -\sin\theta\, d\theta$

$-dw = \sin\theta\, d\theta$

$$= -\frac{1}{5}\int \frac{w^2}{(1-w^2)^2}\,dw$$

$$= -\frac{1}{5}\int \frac{w^2}{(w-1)^2(w+1)^2}\,dw$$

$*$ $\frac{w^2}{(w-1)^2(w+1)^2} = \frac{A}{w-1} + \frac{B}{(w-1)^2} + \frac{C}{w+1} + \frac{D}{(w+1)^2}$

$$w^2 = A(w-1)(w+1)^2 + B(w+1)^2 + C(w+1)(w-1)^2 + D(w-1)^2$$

Let $w=1$ $\quad 1 = B(2^2) \Rightarrow B = \frac{1}{4}$

$w=-1$ $\quad 1 = D(-2)^2 \Rightarrow D = \frac{1}{4}$

$w=2$ $\quad 4 = A(3^2) + \frac{1}{4}(3^2) + C(3) + \frac{1}{4}$

$\frac{3}{2} = 9A + 3C$

$\frac{1}{2} = 3A + C \quad \Rightarrow C = \frac{1}{2} - 3A$

$w=0$ $\quad 0 = -A + \frac{1}{4} + C + \frac{1}{4}$

$-\frac{1}{2} = -A + C \quad \Rightarrow C = -\frac{1}{2} + A$

$\therefore \frac{1}{2} - 3A = -\frac{1}{2} + A$

$1 = 4A \Rightarrow A = \frac{1}{4}$

$C = \frac{1}{2} - 3\left(\frac{1}{4}\right) = -\frac{1}{4}$

$\therefore *$ $\frac{w^2}{(w-1)^2(w+1)^2} = \frac{1}{4}\frac{1}{w-1} + \frac{1}{4}\frac{1}{(w-1)^2} - \frac{1}{4}\frac{1}{w+1} + \frac{1}{4}\frac{1}{(w+1)^2}$

$-\frac{1}{5}\int \frac{w^2}{(w-1)^2(w+1)^2}\,dw = -\frac{1}{20}\int\left[\frac{1}{w-1} + \frac{1}{(w-1)^2} - \frac{1}{w+1} + \frac{1}{(w+1)^2}\right] dw$

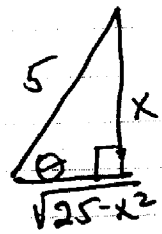
$= -\frac{1}{20}\left[\ln|w-1| - \frac{1}{w-1} - \ln|w+1| - \frac{1}{w+1}\right.$

V. continued,

Since $w = \cos\theta$, the preceeding becomes

$$-\frac{1}{20}\left[\ln|\cos\theta - 1| - \frac{1}{\cos\theta - 1} - \ln|\cos\theta + 1| - \frac{1}{\cos\theta + 1}\right] + C$$

and since $x = 5\sin\theta$, $\sin\theta = \frac{x}{5}$



$$\cos\theta = \frac{\sqrt{25-x^2}}{5}$$

∴ Answer is

$$\boxed{-\frac{1}{20}\left[\ln\left|\frac{\sqrt{25-x^2}}{5} - 1\right| - \frac{1}{\frac{\sqrt{25-x^2}}{5} - 1} - \ln\left|\frac{\sqrt{25-x^2}}{5} + 1\right| - \frac{1}{\frac{\sqrt{25-x^2}}{5} + 1}\right] + C}$$

VI. $\int \frac{3x+2}{x^2+7x+10}\,dx = \int \frac{3x+2}{(x+5)(x+2)}\,dx = \int\left(\frac{A}{x+5} + \frac{B}{x+2}\right)dx$

$$= A\ln|x+5| + B\ln|x+2| + C$$

where
$3x+2 = A(x+2) + B(x+5)$

$x = -5 \quad -13 = A(-3) \Rightarrow A = \frac{13}{3}$

$x = -2 \quad -4 = B(3) : B = -\frac{4}{3}$

$$= \boxed{\frac{13}{3}\ln|x+5| - \frac{4}{3}\ln|x+2| + C}$$