1. $\frac{dy}{dx} + 2xy = 7x$ $\qquad$ $\frac{dy}{dx} = $ [illegible]

$$\mu = e^{\int 2x} = e^{x^2} \qquad = -(2x)\left(y - \frac{7}{2}\right)$$

$$\frac{dy}{dx} + 2xy = 7x$$

$$\frac{d}{dx}\left(y e^{x^2}\right) = 7x e^{x^2}$$

$$y e^{x^2} = \frac{7}{2} e^{x^2} + c$$

$$\boxed{y = \frac{7}{2} + C e^{-x^2}}$$

2. $\underbrace{2x + \cos(xy^3) y^3}_{M} + \underbrace{\left(3xy^2 \cos(xy^3) + 4y^3\right)}_{N} \frac{dy}{dx} = 0$

$$\frac{\partial M}{\partial y} = -\sin(xy^3)\, 3y^2 x \cancel{\frac{dy}{dx}} + \cos(xy^3)\, 3y^2 \cancel{\frac{dy}{dx}}$$

$$\frac{\partial N}{\partial x} = 3y^2 \cos(xy^3) + 3xy^2\left(-\sin(xy^3)\, y^3\right)$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{exact}$$

$$\phi(x,y) = \int \left[2x + \cos(xy^3) y^3\right] dx = x^2 + \sin(xy^3) + g(y)$$

$$\cos(xy^3)\, 3y^2 x + g'(y) = 3xy^2 \cos(xy^3) + 4y^3$$

$$g'(y) = 4y^3$$

$$g(y) = y^4$$

$$\therefore \boxed{\phi(x,y) = x^2 + \sin(xy^3) + y^4 = K}$$

3. $\frac{dy}{dx} = \frac{y^2 - 2x^2}{x^2} = \left(\frac{y}{x}\right)^2 - 2 \; ; \; v = \frac{y}{x} \; ; \; y = vx \; ; \; \frac{dy}{dx} = \frac{dv}{dx} x + v$

$$\frac{dv}{dx} x + v = v^2 - 2 \Rightarrow \int \frac{1}{v^2 - v - 2}\, dv = \int \frac{1}{x}\, dx$$

$$\int \frac{1}{3}\frac{1}{v-2} - \frac{1}{3}\frac{1}{v+1}\, dv = \frac{1}{x} dx \Rightarrow \frac{1}{3} \ln\left|\frac{v-2}{v+1}\right| = \ln x + c$$

$$\frac{v-2}{v+1} = Kx^3 \Rightarrow v - 2 = Kx^3(v+1) \Rightarrow \frac{y}{x} = v = \frac{Kx^3 + 2}{1 - Kx^3} \Rightarrow \boxed{y = x\left(\frac{Kx^3 + 2}{1 - Kx^3}\right)}$$

(6)

x	y	$f(x,y)h$
1	3	$7(.1)$
1.1	3.7	

$f(x,y) = y^2 - 2x^2$

$f(1,3) = 9 - 2 = \cancel{8}\,7$

Answer 3.7

(9) $y(t) =$ account balance

$t =$ years

$y(0) = 1000\,000$

$\frac{dy}{dt} = .05y - 100{,}000$

$\frac{dy}{dt} - .05y = -100\,000$

$\frac{d}{dt}\left(e^{-.05t}y\right) = -100\,000\, e^{-.05t}$

$e^{-.05t}y = \frac{-100000\, e^{-.05t}}{-.05} + C$

$y(t) = 2\,000\,000 + Ce^{+.05t}$

$1000000 = y(0) = 2000000 + c \Rightarrow C = -1000\,000$

$y(t) = 2{,}000{,}000 - 1000{,}000\, e^{+.05t}$

Now solve for t: $y(t) = 0$

$2\,000\,000 = 1000\,000\, e^{+.05t}$

$2 = e^{+.05t}$

$\frac{\ln 2}{+.05} = t \Rightarrow \boxed{t = +20 \ln 2 = 13.86 \text{ years}}$

(1) $\frac{dy}{dx} - y = 2xe^{x}$, $y(0) = 1$

$\mu = e^{-x}$

$\frac{d}{dx}(ye^{-x}) = 2x$

$ye^{-x} = x^2 + C$

$y = e^{x}(x^2 + c)$

$1 = y(0) = e^{0}(0 + c) \Rightarrow c = 1$

$y = e^{x}(x^2 + 1)$

(2) $\underbrace{2xy^3}_{M} + \underbrace{[3x^2y^2 + 2y\cos(y^2)]}_{N}\frac{dy}{dx} = 0$

$\frac{\partial M}{\partial y} = 2x\,3y^2 = 6xy^2$, $\frac{\partial N}{\partial x} = 6xy^2$

exact since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\phi(x,y) = \int 2xy^3\,dx = x^2y^3 + g(y)$

$3x^2y^2 + g'(y) = 3x^2y^2 + 2y\cos(y^2)$

$g'(y) = 2y\cos(y^2) \Rightarrow g(y) = \sin(y^2)$

$\boxed{x^2y^3 + \sin(y^2) = K}$

(3) $\underbrace{2xy^2\cos(x^2y^3)}_{M} + \underbrace{[3x^2y\cos(x^2y^3) + 2]}_{N}\frac{dy}{dx} = 0$

$\frac{\partial M}{\partial y} = 4xy\cos(x^2y^3) + 2xy^2(-\sin(x^2y^3))\,x^2 3y^2$

$= 4xy\cos(x^2y^3) - 6x^3y^4\sin(x^2y^3)$

$\frac{\partial N}{\partial x} = 6xy\cos(x^2y^3) + 3x^2y(-\sin(x^2y^3))\,2y^3x$

$= 6xy\cos(x^2y^3) - 6x^3y^4(\sin(x^2y^3))$

$\frac{d\ln\mu}{dy} = \frac{N_x - M_y}{M} = \frac{2xy\cos(x^2y^3)}{2xy^2\cos(x^2y^3)} = \frac{1}{y}$

$\ln\mu = \ln y \Rightarrow \mu = y$

$2xy^3\cos(x^2y^3) + [3x^2y^2\cos(x^2y^3) + 2y]\frac{dy}{dx}$ is exact

$\phi(x,y) = \int 2xy^3\cos(x^2y^3)\,dx = \sin(x^2y^3) + g(y)$

$\cos(x^2y^3)\,3x^2y^2 + g'(y) = 3x^2y^2\cos(x^2y^3) + 2y$

$g'(y) = 2y \Rightarrow g(y) = y^2$

$\therefore \boxed{\sin(x^2y^3) + y^2 = K}$

⑥ $\frac{dy}{dx} - \frac{2}{x} y = -x y^3$ $\qquad v = y^{1-3} = y^{-2}$

$\frac{dv}{dx} = (-2) y^{-3} \frac{dy}{dx}$

$\frac{dy}{dx} y^{-3} - \frac{2}{x} \frac{1}{y^2} = -x$

$-\frac{1}{2} \frac{dv}{dx} - \frac{2}{x} v = -x$

$\frac{dv}{dx} + \frac{4}{x} v = 2x$

$\mu = e^{4 \ln x} = x^4$

$\frac{d}{dx}(x^4 v) = 2x^5$

$x^4 v = \frac{2}{6} x^6 + C \Rightarrow v = \frac{1}{3} x^2 + C x^{-4}$

$\frac{1}{y^2} = \frac{1}{3} x^2 + C x^{-4}$

$y^2 = \frac{1}{\frac{1}{3} x^2 + C x^{-4}}$

④ $\frac{dy}{dx} = \frac{x^2 \tan y + \tan y}{x^3 + x}$

$= \frac{(x^2+1) \tan y}{x(x^2+1)} = \frac{\tan y}{x}$

$\int \frac{\cos y}{\sin y} \, dy = \int \frac{1}{x} \, dx$

$\ln|\sin y| = \ln|x| + C$

⑤ $\frac{dy}{dx} = \frac{y \ln y - y \ln x}{x} = \frac{y(\ln y - \ln x)}{x}$

$\frac{dy}{dx} = \frac{y}{x} \ln \frac{y}{x} = v \ln v$ where $v = \frac{y}{x}$

$y = vx; \ \frac{dy}{dx} = \frac{dv}{dx} x + v$

$\frac{dv}{dx} x + v = v \ln v$

$\frac{dv}{dx} x = v \ln v - v$

$\frac{1}{v \ln v - v} \, dv = \frac{1}{x} \, dx$

$\int \frac{1}{v(\ln v - 1)} \, dv = \int \frac{1}{x} \, dx$

$\ln(|\ln v - 1|) = \ln|x| + C$

$\ln\left|\ln \frac{y}{x} - 1\right| = \ln|x| + C$

(7) $\frac{dy}{dx} = (y-1)(y-3)^2(y-5)$

equilibrium solutions

$y(0) = 0$

$y(0) = 2$

$y(0) = 4$

$y(0) = 6.$

$z = (y-1)(y-3)^2(y-5)$