① $\frac{dy}{dx} = \frac{y^2}{yx^2+y}$

$\frac{dy}{dx} = \frac{y^2}{y(x^2+1)} = \frac{y}{x^2+1}$

$\int \frac{1}{y}\,dy = \int \frac{1}{x^2+1}\,dx$ ③

$\ln|y| = \arctan(x) + c$ ④

$|y| = e^{\arctan(x)+c}$ ⑤

$|y| = e^{\arctan(x)} e^{c}$ ⑥

$y = Ae^{\arctan(x)}$ ⑦

check: $\frac{dy}{dx} = Ae^{\arctan(x)}\frac{1}{x^2+1}$

$= \frac{Ae^{\arctan(x)}}{x^2+1}$

$= \frac{y}{x^2+1}$

② $\frac{dy}{dx} = \frac{-y}{2x} + x^{3/2} - x^{1/2}$

$\frac{dy}{dx} + \frac{1}{2x}y = x^{3/2} - x^{1/2}$

③ $I(x) = e^{\int \frac{1}{2}\frac{1}{x}dx} = e^{\frac{1}{2}\ln x} = x^{\frac{1}{2}}$

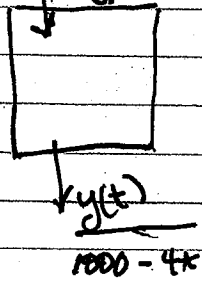
④ $\frac{d}{dx}(yx^{\frac{1}{2}}) = x^{\frac{1}{2}}(x^{3/2} - x^{\frac{1}{2}}) = x^2 - x$

⑤ $yx^{\frac{1}{2}} = \frac{1}{4}x^4 - \frac{1}{2}x^2 + C$

⑥ $y = \frac{1}{4}x^{7/2} - \frac{1}{2}x^{3/2} + Cx^{-\frac{1}{2}}$

③. $y(t)$ = amount of toxic substance

$y(0) = 5$



$\frac{dy}{dt} = 0 - \frac{y(t)}{1000-4t}\,2$

$= \frac{-6y}{250-t}$

$\frac{dy}{dt} + \frac{6}{250-t}y = 0$

$I(t) = e^{-6\ln(250-t)} = (250-t)^{-6}$

$\frac{d}{dt}\left((250-t)^{-6}y\right) = 0$

$(250-t)^{-6}y = K$

$y = K(250-t)^6$

$5 = K(250^6) \Rightarrow K = 5(250^{-6})$

$y(t) = 5(250^{-6})(250-t)^6 = 5\left(1-\frac{t}{250}\right)^6$

Solve for t, $y(t) = 5\left(1-\frac{t}{250}\right)^6 = 2$

$\left(1-\frac{t}{250}\right)^6 = .4$

$1 - \frac{t}{250} = .4^{1/6} \Rightarrow t = 250\left(1-.4^{1/6}\right) \approx 35.4$ min.