Section 5.2 Problem.

$$\frac{d^2y}{dt^2} + \frac{3t}{1+t^2}\frac{dy}{dt} + \frac{1}{1+t^2}y = 0 \quad , \quad \begin{array}{l} y(0) = 2 \\ y'(0) = 3 \end{array}$$

$$(1+t^2)\frac{d^2y}{dt^2} + 3t\frac{dy}{dt} + y = 0$$

$$(1+t^2)\sum_{n=0}^{\infty} n(n-1)a_n t^{n-2} + 3t\sum_{n=0}^{\infty} n a_n t^{n-1} + \sum_{n=0}^{\infty} a_n t^n$$

$$\cdots = \sum_{n=0}^{\infty} n(n-1)a_n t^{n-2} + \sum_{n=0}^{\infty}[n(n-1) + 3n + 1]a_n t^n$$

$$= \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2} t^n + \sum_{n=0}^{\infty}(n+1)^2 a_n t^n$$

$$(n+2)(n+1)a_{n+2} + (n+1)^2 a_n = 0$$

$$a_{n+2} = \frac{-(n+1)^2}{(n+2)(n+1)}a_n = \frac{-(n+1)}{n+2}a_n$$

(i) $a_0 = 1$ $\quad a_2 = -\frac{a_0}{2} = -\frac{1}{2}$

$a_1 = 0$

$a_{2k+1} = 0$ $\quad a_4 = -\frac{3}{4}a_2 = \frac{1}{2}\cdot\frac{3}{4}$

$$a_6 = -\frac{5a_4}{6} = -\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}$$

inductively

$$a_{2n} = (-1)^n\frac{1\cdot 3\cdots(2n-1)}{2\cdot 4\cdots 2n} = (-1)^n\frac{1\cdot 3\cdots(2n-1)}{2^n n!}$$

$$y_1(t) = \sum_{n=0}^{\infty}(-1)^n\frac{1\cdot 3\cdots(2n-1)}{2^n n!}t^{2n}$$

Note: $y_1(0) = 1$, $y_1'(0) = 0$

Exercise 1 show convergence for $|t| < 1$

(ii) $a_0 = 0$ $\quad$ Exercise 2 $\quad y_2(t) = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n n!}{3\cdot 5\cdots(2n+1)}t^{2n+1}$ $\quad y_2(0) = 0$

$a_1 = 1$ $\quad y_2'(0) = 1$

$a_{2n} = 0$ & show radius of convergence

$$\therefore y(t) = 2y_1(t) + 3y_2(t)$$

Assignment: Read 5.1, 5.2
Do problems 2 & 8 on page 259.