(13) $w = \sqrt[3]{x}$; $w^3 = x$; $dx = 3w^2\,dw$

$$* \int e^{\sqrt[3]{x}}\,dx = \int e^w 3w^2\,dw = 3\int w^2 e^w\,dw$$

Use integration by parts twice to find

$$\int w^2 e^w\,dw = w^2 e^w - \int e^w 2w\,dw$$

$u = w^2$ $\quad dv = e^w dw$

$du = 2w\,dw$ $\quad v = e^w$

$$= w^2 e^w - 2\int w e^w\,dw$$

$$\int w e^w\,dw = w e^w - \int e^w\,dw$$

$u = w$ $\quad dv = e^w dw$

$du = dw$ $\quad v = e^w$

$$= w e^w - e^w$$

$$\therefore * \int e^{\sqrt[3]{x}}\,dx = 3\int w^2 e^w$$

$$= 3\left[w^2 e^w - 2\int w e^w\,dw\right]$$

$$= 3\left[w^2 e^w - 2(w e^w - e^w)\right] + C$$

$$= 3w^2 e^w - 6 w e^w + 6 e^w + C$$

$$= 3x^{2/3} e^{x^{1/3}} - 6x^{1/3} e^{x^{1/3}} + 6 e^{x^{1/3}} + C$$

$$= \boxed{\left[3x^{2/3} - 6x^{1/3} + 6\right] e^{x^{1/3}} + C}$$

(14) $\int \frac{x^2+2}{x+2} = \int \left(x - 2 + \frac{6}{x+2}\right) dx = \frac{1}{2}x^2 - 2x + 6\ln|x+2| + C$

$$\begin{array}{r|l} & x - 2 \\ \hline x+2 & x^2 + 2 \\ & x^2 + 2x \\ \hline & -2x + 2 \\ & -2x - 4 \\ \hline & 6 \end{array}$$

(15) $\frac{x-1}{x^2+2x} = \frac{x-1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$x-1 = A(x+2) + Bx = (A+B)x + 2A$

$2A = -1 \qquad A+B = 1$

$A = -\frac{1}{2} \qquad B = 1 - A = 1 + \frac{1}{2} = \frac{3}{2}$

$\int \frac{x-1}{x^2+2x}\,dx = \int \left(-\frac{1}{2}\frac{1}{x} + \frac{3}{2}\frac{1}{x+2}\right)dx$

$= \boxed{-\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x+2| + C}$

16) $\int \frac{x+1}{9x^2+6x+5}\,dx = \int \frac{x+1}{(3x+1)^2+2^2}$

$w = 3x+1$
$w-1 = 3x$
$\frac{1}{3}(w-1) = x$
$dw = 3\,dx$
$\frac{1}{3}\,dw = dx$

$= \int \frac{\frac{1}{3}(w-1)+1}{w^2+2^2}\,\frac{1}{3}\,dw$

$= \frac{1}{9}\int \frac{w-1+3}{w^2+2^2}\,dw$

$= \frac{1}{9}\int \frac{w+2}{w^2+2^2}\,dw = \frac{1}{9}\int\left(\frac{1}{2}\frac{2w}{w^2+2^2} + \frac{2}{w^2+2^2}\right)$ dw

$= \frac{1}{9}\left[\frac{1}{2}\ln(w^2+4) + 2\cdot\frac{1}{2}\arctan\frac{w}{2}\right] + C$

$= \boxed{\frac{1}{18}\ln(9x^2+6x+5) + \frac{1}{9}\arctan\frac{3x+1}{2} + C}$

(20) $\int \tan^5\theta\sec^3\theta\,d\theta = \int \tan^4\theta\sec^2\theta\ \sec\theta\tan\theta\,d\theta$

$= \int (\sec^2\theta-1)^2\sec^2\theta\ \sec\theta\tan\theta\,d\theta$

$= \int (w^2-1)^2 w^2\,dw$

$w = \sec\theta$
$dw = \sec\theta\tan\theta\,d\theta$

$= \int (w^4-2w^2+1)w^2\,dw$

$= \int (w^6-2w^4+w^2)\,dw$

$= \frac{1}{7}w^7 - \frac{2}{5}w^5 + \frac{1}{3}w^3 + C$

$= \boxed{\frac{1}{7}\sec^7\theta - \frac{2}{5}\sec^5\theta + \frac{1}{3}\sec^3\theta + C}$

(21) $\int \frac{1}{\sqrt{x^2-4x}}\,dx = \int \frac{1}{\sqrt{x^2-4x+4-2^2}}\,dx = \int \frac{1}{\sqrt{(x-2)^2-2^2}}\,dx$

$w = x-2$
$dw = dx$

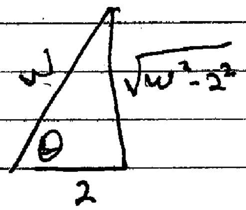
$= \int \frac{1}{\sqrt{w^2-2^2}}\,dw$

$w = \cancel{\sec}\ 2\sec\theta$
$\sqrt{w^2-2^2} = \sqrt{2^2(\sec^2\theta-1)}$
$= 2\tan\theta$

$dw = 2\sec\theta\tan\theta\,d\theta$

$= \int \frac{1}{2\tan\theta}\,2\sec\theta\tan\theta\,d\theta$

$= \int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta| + C$



$= \ln\left|\frac{w}{2} + \frac{\sqrt{w^2-2^2}}{2}\right| + C$

$= \ln\left|\frac{x-2}{2} + \frac{\sqrt{x^2-4x}}{2}\right| + C$

(22) $x = \sqrt{t}$ $\quad t = x^2$ $\quad dt = 2x\,dx$

$\int t e^{\sqrt{t}}\,dt = \int x^2 e^x\, 2x\,dx = 2\int x^3 e^x\,dx$

$\int x^3 e^x\,dx = x^3 e^x - \int e^x\, 3x^2\,dx$

$= x^3e^x - 3\int x^2 e^x\,dx$

$= x^3e^x - 3\left[x^2e^x - \int e^x\,2x\,dx\right]$

$= x^3e^x - 3x^2e^x + 6\int xe^x\,dx$

$= x^3e^x - 3x^2e^x + 6\left[xe^x - \int e^x\,dx\right]$

$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

$\therefore\ 2\int e^x x^3\,dx = 2e^x\left[x^3 - 3x^2 + 6x - 6\right] + C = \boxed{2e^{\sqrt{t}}\left[t^{3/2} - 3t + 6\sqrt{t} - 6 \ldots\right.}$

(23) $\int \frac{1}{x\sqrt{x^2+1}}\,dx \qquad = \int \frac{1}{\tan\theta\,\sec\theta}\,\sec^2\theta\,d\theta$

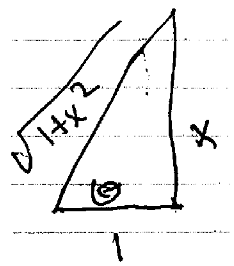
$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$= \int \frac{\sec\theta}{\tan\theta}\,d\theta$

$\sqrt{x^2+1} = \sqrt{\tan^2\theta+1} = \sec\theta$

$= \int \frac{\sec\theta\,\cos\theta}{\sin\theta}\,d\theta$



$= \int \csc\theta\,d\theta = \ln|\csc\theta - \cot\theta| + C$

$= \boxed{\ln\left|\frac{\sqrt{x^2+1}}{x} - \frac{1}{x}\right| + C}$