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(4) $\int_0^{\pi/6} t \sin 2t \, dt = uv - \int_0^{\pi/6} v \, du = -\frac{1}{2} t \cos 2t \Big|_0^{\pi/6} - \int_0^{\pi/6} (-\frac{1}{2}\cos 2t) \, dt$

$u = t \qquad dv = \sin 2t \, dt$

$du = dt \qquad v = -\frac{1}{2}\cos 2t$

$= \left(-\frac{1}{2} t \cos 2t + \frac{1}{4}\sin 2t\right)\Big|_0^{\pi/6}$

$= -\frac{1}{2}\,\pi/6 \cos\frac{\pi}{3} + \frac{1}{4}\sin\frac{\pi}{3} - (0+0)$

$= \boxed{-\frac{\pi}{24} + \frac{1}{8}\sqrt{3}}$

(5) $\int \frac{1}{2t^2+3t+1} \, dt = \int \frac{1}{(2t+1)(t+1)} \, dt$

$= \int \left(\frac{A}{2t+1} + \frac{B}{t+1}\right) dt$

$1 = A(t+1) + B(2t+1)$

$t=-1 \quad 1 = B(-1) \Rightarrow B = -1$

$t = \frac{-1}{2} \quad 1 = A(\frac{1}{2}) \Rightarrow A = 2$

$= \int \left(\frac{2}{2t+1} - \frac{1}{t+1}\right) dt$

$= \boxed{\ln|2t+1| - \ln|t+1| + C}$

(6) $\int_1^2 x^5 \ln x \, dx = \frac{1}{6}x^6 \Big|_1^2 - \int_1^2 \frac{1}{6}x^6 \frac{1}{x} \, dx$

$u = \ln x \qquad dv = x^5 \, dx$

$du = \frac{1}{x} dx \qquad v = \frac{1}{6}x^6$

$= \frac{1}{6}x^6 \ln x \Big|_1^2 - \int_1^2 \frac{1}{6}x^5 \, dx$

$= \left(\frac{1}{6}(\ln x)x^6 - \frac{1}{36}x^6\right)\Big|_1^2$

$= \frac{1}{6}2^6 \ln 2 - \frac{1}{36}2^6 - (0 - \frac{1}{36})$

$= \frac{32}{3}\ln 2 - \left[\frac{1}{36}\right]\left[2^6 - 1\right]$

$= \frac{32}{3}\ln 2 - \left[\frac{1}{36}\right]\left[63\right] = \boxed{\frac{32}{3}\ln 2 - \frac{7}{4}}$

(7) $\int_0^{\pi/2} \sin^3\theta \cos^2\theta \, d\theta = \int_0^{\pi/2} (1-\cos^2\theta)\cos^2\theta \sin\theta \, d\theta$

$u = \cos\theta$

$-du = \sin\theta \, d\theta$

$= -\int_1^0 (1-u^2)u^2 \, du = -\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)\Big|_1^0$

$= \frac{1}{3} - \frac{1}{5} = \boxed{\frac{2}{15}}$

(11) $x = \sec\theta \qquad dx = \sec\theta\tan\theta\, d\theta \qquad \sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = \tan\theta$

$$\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx = \int_0^{\pi/3} \frac{\tan\theta}{\sec\theta}\sec\theta\tan\theta = \int_0^{\pi/3}\tan^2\theta\, d\theta$$

$$= \int_0^{\pi/3}(\sec^2\theta - 1)\,d\theta = (\tan\theta - \theta)\Big|_0^{\pi/3} = \boxed{\sqrt{3} - \frac{\pi}{3}}$$

(16) $$\int \frac{\sec^6\theta}{\tan^2\theta}\,d\theta = \int \frac{(\tan^2\theta+1)^2}{\tan^2\theta}\sec^2\theta\, d\theta = \int \frac{(u^2+1)^2}{u^2}\,du$$

$u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$$= \int\left(u^2 + 2 + \frac{1}{u^2}\right)du = \frac{1}{3}u^3 + 2u - \frac{1}{u} + C$$

$$= \boxed{\left(\frac{1}{3}\tan^3\theta + 2\tan\theta - \cot\theta\right) + C}$$

(17) $$\int x\sec x\tan x\, dx = x\sec x - \int \sec x\, dx$$

$$= \boxed{x\sec x - \ln|\sec x + \tan x| + C}$$

$u = x \qquad dv = \sec x\tan x\, dx$

$du = dx \qquad v = \sec x$

(18) $$\frac{x^2+8x-3}{x^3+3x^2} = \frac{x^2+8x-3}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

$$x^2+8x-3 = Ax(x+3) + B(x+3) + Cx^2$$

$x=0 \qquad -3 = B(3) \Rightarrow B = -1$

$x=-3 \qquad 9-24-3 = -18 = C(9) \Rightarrow C = -2$

$A + C = 1$

$A = 1 - C = 1-(-2) = 3$

$$\int\left(\frac{3}{x} - \frac{1}{x^2} + \frac{-2}{x+3}\right)dx = \boxed{3\ln|x| + \frac{1}{x} - 2\ln|x+3| + C}$$