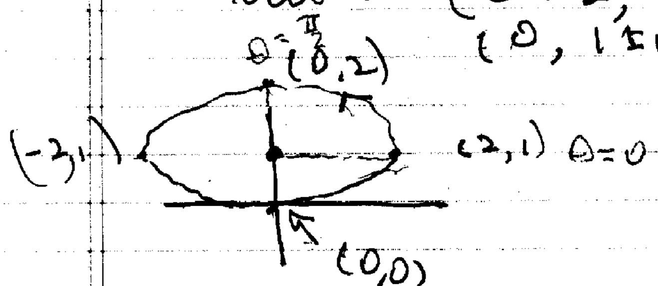
Chapter Ten HW

(4) $x = 2\cos\theta \quad y = 1 + \sin\theta$

$$\left(\frac{x}{2}\right)^2 + (y-1)^2 = \cos^2\theta + \sin^2\theta$$

$$\frac{x^2}{2^2} + \frac{(y-1)^2}{1^2} = 1$$

ellipse centered at $(0, 1)$
vertices $(0 \pm 2, 1)$
$(0, 1 \pm 1)$



horizontal tan line $\frac{dy}{d\theta} = \cos\theta = 0$

$\theta = \frac{\pi}{2} \quad (x,y) = (0,2)$

$\theta = \frac{3\pi}{2} \quad (x,y) = (0,0)$

vertical tan lines $\frac{dy}{d\theta} = -2\sin\theta = 0$

$\theta = 0 \quad (x,y) = (2,1)$

$\theta = \pi \quad (x,y) = (-2,1)$

$$A = 4\int_0^{\frac{\pi}{2}} (y-1)\frac{dx}{d\theta}\,d\theta = 4\int_0^{\frac{\pi}{2}} (1+\sin\theta-1)(-2\sin\theta)\,d\theta$$

$$= -8\int_0^{\frac{\pi}{2}} [\sin^2\theta]\,d\theta$$

$$= 8\int_0^{\frac{\pi}{2}} \sin^2\theta\,d\theta = 8\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$ and finish

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\,d\theta = \int_0^{2\pi} \sqrt{(-2\sin\theta)^2 + (\cos\theta)^2}\,d\theta$$

then numerically integrate.

$x = r\cos\theta$

$y = r\sin\theta$

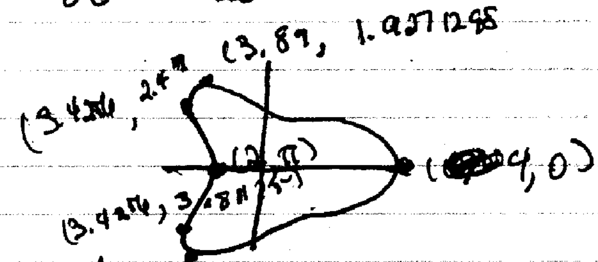
$r = f(\theta)$

horizontal tan lines $\quad 0 = \dfrac{dy}{d\theta} = \dfrac{dr}{d\theta}\sin\theta + r\cos\theta$

vertical tan lines $\quad 0 = \dfrac{dx}{d\theta} = \dfrac{dr}{d\theta}\cos\theta - r\sin\theta$

#12 $\quad r = 3 + \cos 3\theta$

$\dfrac{dr}{d\theta} = -\sin(3\theta)(3)$



horizontal tan lines $\quad 0 = \dfrac{dy}{d\theta} = -\sin(3\theta)3\sin\theta + (3+\cos 3\theta)\cos\theta$

$0 = -3\sin\theta\sin(3\theta) + 3\cos\theta + \cos\theta\cos 3\theta$

$\theta = 4.356$ radians

$\theta = 1.927298$ radians

vertical tan line

$0 = \dfrac{dx}{d\theta} = -\sin(3\theta)\,3\cos\theta - (3+\cos 3\theta)\sin\theta$

$\theta = 0$

$\theta = 2.47145\,18$

$\theta = \pi$

$\theta = 3.8117335$

$\theta = 2\pi$

	θ	r
vertical	0	4
vertical	2.471458	3.4256
horizontal	1.927298	3.8267
vertical	π	2
vertical	3.811735	3.4256
horizontal	4.356	3.8768
vertical	2π	0

$\text{Area} = \dfrac{1}{2}\displaystyle\int_0^{2\pi} r^2\, d\theta$

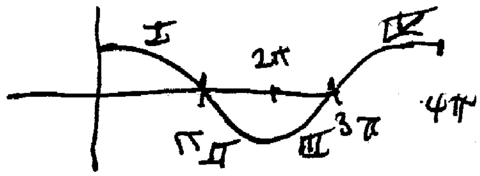
$= \dfrac{1}{2}\displaystyle\int_0^{2\pi} (3 + \cos 3\theta)^2\, d\theta$ and finish

$\text{length} = \displaystyle\int_0^{2\pi} \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}\, d\theta$

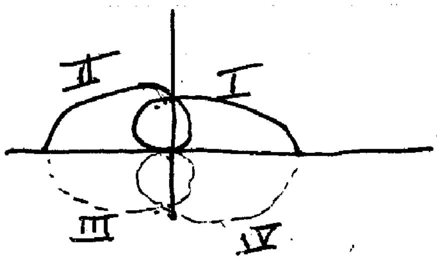
$= \displaystyle\int_0^{2\pi} \sqrt{(3+\cos 3\theta)^2 + (3\sin 3\theta)^2}\, d\theta$

(14) $z = 2\cos\left(\frac{t}{2}\right)$

period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$



$r = 2\cos\left(\frac{\theta}{2}\right)$



$\frac{dr}{d\theta} = -2\sin\left(\frac{\theta}{2}\right)\frac{1}{2} = -\sin\left(\frac{\theta}{2}\right)$

horizontal tan line $\quad 0 = \frac{dy}{d\theta} = -\sin\left(\frac{\theta}{2}\right)\sin\theta + 2\cos\left(\frac{\theta}{2}\right)\cos\theta$

$\theta = 1.23$
$\theta = \pi$
$\theta = 5.0522$
$\theta = 7.514$
$\theta = 3\pi$
$\theta = 11.3354$

vertical tan line

$0 = \frac{dx}{d\theta} = -\sin\left(\frac{\theta}{2}\right)\cos\theta - 2\cos\frac{\theta}{2}\sin\theta$

$\theta = 0$
$\theta = 2.30$
$\theta = 3.98$
$\theta = 2\pi$
$\theta = 8.58$
$\theta = 10.266$
$\theta = 4\pi$

(16) $r = \dfrac{\frac{3}{2}}{1-\cos\theta}$ ~~Ellipse~~ Parabola with focus at the origin.

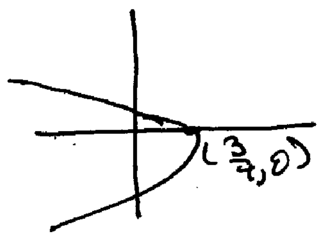
$r - r\cos\theta = \frac{3}{2}$

$r^2 = \left(\frac{3}{2} - r\cos\theta\right)^2$

$\cancel{x^2} + y^2 = \left(\frac{3}{2} - x\right)^2 = \frac{9}{4} - 3x + \cancel{x^2}$

$y^2 = \frac{9}{4} - 3x = -3\left(x - \frac{3}{4}\right)$

$x - \frac{3}{4} = -\frac{1}{3} y^2$



$-\frac{1}{3} = -\frac{1}{4p}$

$3 = 4p$

$p = \frac{3}{4}$

No horizontal tan line

vertical tan line at $(x,y) = \left(\frac{3}{4}, 0\right)$.

(46) $4x^2 - y^2 = 16$

$\frac{x^2}{4} - \frac{y^2}{16} = 1$

$\frac{x^2}{2^2} - \frac{y^2}{4^2} = 1$

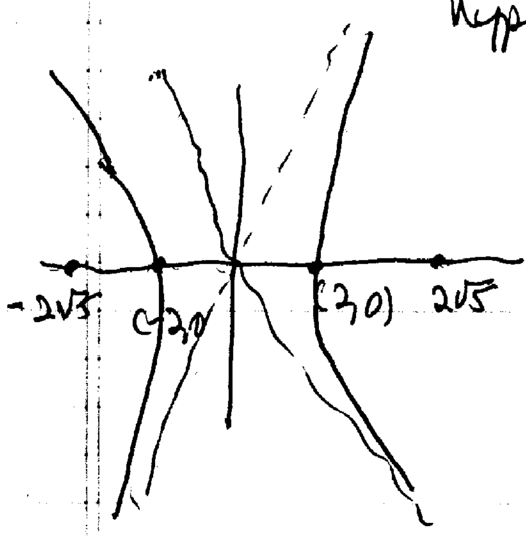
vertices $(\pm 2, 0)$

$c^2 = 4 + 16 = 20$

$c = \pm 2\sqrt{5}$

foci $(\pm 2\sqrt{5}, 0)$

Hyperbola



$\frac{x^2}{2^2} - \frac{y^2}{4^2} = 0$

$\frac{y^2}{4^2} = \frac{x^2}{2^2}$

$\frac{y}{4} = \pm \frac{x}{2}$

$y = \pm 2x$

(48) $25x^2 + 4y^2 + 50x - 16y = 59$

$25(x^2 + 2x) + 4(y^2 - 4y) = 59$

$25(x^2 + 2x + 1) + 4(y^2 - 4y + 4) = 59 + 25 + 16 = 10^2$

$$\frac{(x+1)^2}{2^2} + \frac{(y-2)^2}{5^2} = 1$$

ellipse center at $(-1, 2)$

vertices ~~(-1±5, 2)~~ $(-1, 2\pm 5)$

$(-1 \pm 2, 2)$

foci $c^2 = 5^2 - 2^2 = 25 - 4 = 21$

$c = \pm\sqrt{21}$

foci $(-1, 2 \pm \sqrt{21})$