Spring, 2015 Name: Key

MAT 162002 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book. When you turn in your test, staple your notes to this sheet.

For 1-4, calculate the following:

10

1. $\int x^2 \cos(5x+1)\ dx$

$u = x^2 \quad dv = \cos(5x+1)\,dx$

$du = 2x\,dx \quad v = \frac{\sin(5x+1)}{5}$ 2

2

$= uv - \int v\,du = \frac{1}{5}x^2\sin(5x+1) - \int \frac{\sin(5x+1)}{5}\,2x\,dx$

$= \frac{1}{5}x^2\sin(5x+1) - \frac{2}{5}\int x\sin(5x+1)\,dx$ 1

$u = x \quad dv = \sin(5x+1)\,dx$

$du = dx \quad v = -\frac{\cos(5x+1)}{5}$ 2

2

$= \frac{1}{5}x^2\sin(5x+1) - \frac{2}{5}\left[-\frac{x\cos(5x+1)}{5} + \frac{1}{5}\int \cos(5x+1)\,dx\right]$

$= \frac{1}{5}x^2\sin(5x+1) + \frac{2}{25}x\cos(5x+1) - \frac{2\sin(5x+1)}{125} + C$ 1

2. $\int \tan^3(x)\sec^5(x)\,dx$

10

$\int \tan^2 x\sec^4 x\,\tan x\sec x\,dx = \int (\sec^2 x - 1)\sec^4 x\,\tan x\sec x\,dx$

$u = \sec x$

$du = \sec x\tan x\,dx$

$= \int (u^2-1)u^4\,du = \int (u^6 - u^4)\,du$

$= \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C}$

10

3. $\int x^3\sqrt{9-x^2}\,dx$

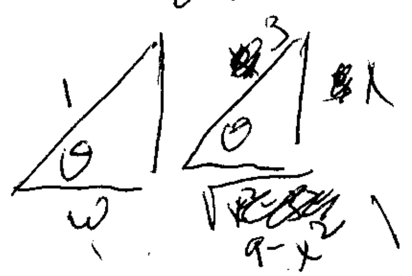
$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$\sqrt{9-x^2} = 3\cos\theta$ 2

$w = \cos\theta$

$-dw = \sin\theta\,d\theta$



$= \int 3^3\sin^3\theta\ 3\cos\theta\ 3\cos\theta\,d\theta$ 1

$= 3^5\int (1-\cos^2\theta)\cos^2\theta\,\sin\theta\,d\theta$ 1

$= -3^5\int (1-w^2)w^2\,dw$ 1

$= -3^5\int (w^2 - w^4)\,dw$

$= -3^5\left[\frac{1}{3}w^3 - \frac{1}{5}w^5\right] + C$

$= -3^5\left[\frac{1}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta\right] + C$ 1

$= -3^5\left[\frac{1}{3}\left(\frac{\sqrt{9-x^2}}{3}\right)^3 - \frac{1}{5}\left(\frac{\sqrt{9-x^2}}{3}\right)^5\right] + C$ 1

4. $\int \frac{3x+5}{x^2+9x+20}dx = -7\int \frac{1}{x+4}dx + 10\int \frac{1}{x+5}dx = \boxed{-7\ln|x+4| + 10\ln|x+5| + C}$ 2 2

no

$(x+4)(x+5)$

$\frac{3x+5}{(x+4)(x+5)} = \frac{A}{x+4} + \frac{B}{x+5} = -7\frac{1}{x+4} + 10\frac{1}{x+5}$

$3x+5 = A(x+5) + B(x+4)$

$-7 = A(1) \Rightarrow A = -7$ 2

$-10 = B(-1) \Rightarrow B = 10$

5. Estimate $\int_2^6 e^{1+\arctan(x)}dx$ using the Midpoint Rule with n = 4. Write the sum; you do not have to evaluate the sum.

no

$\Delta x = \frac{6-2}{4} = 1$ 2

$1(f(2.5) + f(3.5) + f(4.5) + f(5.5))$ 1

4

2 3 4 5 6

6. Calculate the following; it the integral does not converge, state "does not converge."

2

a. $\int_1^{+\infty} x^2e^{-x^3}dx = \lim_{b\to+\infty}\int_1^b x^2e^{-x^3}dx = \lim_{b\to+\infty} -\frac{1}{3}e^{-x^3}\Big|_1^b$ 2 2

8

$= \lim_{b\to+\infty} -\frac{1}{3}e^{-b^3} + \frac{1}{3}e^{-1} = \boxed{\frac{1}{3}e^{-1}}$ converges 2

b. $\int_1^5 \frac{4x}{\sqrt{25-x^2}}dx = \lim_{b\to5^-}\int_1^b \frac{4x}{\sqrt{25-x^2}}dx = \lim_{b\to5^-} -4(25-x^2)^{\frac{1}{2}}\Big|_1^b$ 2 2

8

$= \lim_{b\to5^-} -4(25-b^2)^{\frac{1}{2}} + 4(24)^{\frac{1}{2}} = 4(24)^{\frac{1}{2}}$ 2

1 b 5

7. Tell why the following converge or diverge:

(7)

a. $\int_1^{+\infty}\frac{x^3+7}{(x^8+24)^{0.5}}dx$

3

$\frac{x^3}{5x^4} = \frac{x^3}{(x^8+24x^8)^{\frac{1}{2}}} \le \frac{x^3+7}{(x^8+24)^{\frac{1}{2}}}$

$\frac{1}{5}\int_1^{+\infty}\frac{1}{x}dx$ diverges $\Rightarrow \int_1^{+\infty}\frac{x^3+7}{(x^8+24)^{\frac{1}{2}}}$ diverges 2

2

(6)

b. $\int_1^{+\infty}\frac{7}{(x^4+24)^{0.5}}dx$

3

$\frac{7}{(x^4+24)^{\frac{1}{2}}} \le \frac{7}{x^2}$

$\int_1^{+\infty}\frac{7}{x^2}dx$ converges 2 $\Rightarrow \int_1^{+\infty}\frac{7}{(x^4+24)^{.5}}dx$ converges 2

8. Calculate $\int \frac{7x}{x^2+4x+20}dx$ $= \int \frac{7x}{(x+2)^2+4^2}dx$ 2

10

$= \int \frac{7(x+2)-14}{(x+2)^2+4^2}dx$ 1

$= \frac{7}{2}\int \frac{2(x+2)}{(x+2)^2+4^2}dx - 14\int \frac{1}{(x+2)^2+4^2}dx$

$= \boxed{\frac{7}{2}\ln(x^2+4x+20) - 14\,\frac{1}{4}\arctan\left(\frac{x+2}{4}\right)+C}$

3 3 1

9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the integral): $\int \frac{4x+5}{(x^2+6x+8)^3(x^2+8x+25)^2}dx$

10

$\frac{4x+5}{(x+2)^3(x+4)^3\,((x+4)^2+3^2)^2}$ 2

$= \frac{A_1}{x+2} + \frac{B_1}{(x+2)^2} + \frac{C_1}{(x+2)^3} + \frac{A_2}{(x+4)} + \frac{B_2}{(x+4)^3} + \frac{C_2}{(x+4)^3}$

$+ \frac{C_1x+D_1}{x^2+8x+25} + \frac{C_2x+D_2}{x^2+8x+25}$

2 2 2 2