Spring, 2015 Name: Key

MAT 162001 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book. When you turn in your test, staple your notes to this sheet.

For 1-4, calculate the following:

10 1. $\int x^2\cos(5x)\,dx$ $= uv - \int v\,du = \frac{1}{5}x^2\sin(5x) - \int \frac{1}{5}\sin(5x)\,2x\,dx$ 2

2 $u = x^2$ $dv = \cos(5x)\,dx$

$du = 2x\,dx$ $v = \frac{1}{5}\sin(5x)$

$= \frac{1}{5}x^2\sin(5x) - \frac{2}{5}\int x\sin(5x)\,dx$ 1

$\int x\sin(5x)\,dx$

$= \frac{1}{5}x^2\sin(5x) - \frac{2}{5}\left[-\frac{x\cos 5x}{5} + \frac{1}{5}\int\cos 5x\,dx\right]$ 2

$u = x$ $dv = \sin(5x)\,dx$

$du = dx$ $v = -\frac{\cos(5x)}{5}$ 2

$= \frac{1}{5}x^2\sin(5x) + \frac{2}{25}x\cos(5x) - \frac{2}{125}\sin 5x + C$ ✓ 1

10 2. $\int\tan^3(x)\sec^3(x)x$ $= \int\tan^2x\sec^2x\tan x\sec x\,dx$ 2

$= \int(\sec^2x - 1)\sec^2x\tan x\sec x\,dx$ 2

2 $w = \sec x$

$dw = \sec x\tan x\,dx$

$= \int(w^2-1)w^2\,dw$ 2

$= \frac{1}{5}w^5 - \frac{1}{3}w^3 + C = \frac{1}{5}\sec^5x - \frac{1}{3}\sec^3x + C$ ✓ 2

10 3. $\int x^3\sqrt{4-x^2}\,dx$ $= \int 2^3\sin^3\theta\, 2^2\cos^2\theta\,d\theta$ 1

$= 2^5\int(1-\cos^2\theta)\cos^2\theta\sin\theta\,d\theta$ 1

2 $x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$\sqrt{4-x^2} = 2\cos\theta$

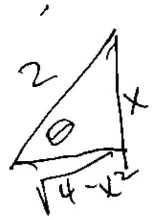
$= 2^5(-1)\int(1-w^2)w^2\,dw$ 1

1 $w = \cos\theta$

$-dw = \sin\theta\,d\theta$

$= -2^5\left(\frac{1}{3}w^3 - \frac{1}{5}w^5\right) + C$ 1

$= -2^5\left(\frac{1}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta\right) + C$ 1

1 


$= -2^5\left(\frac{1}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3 - \frac{1}{5}\left(\frac{\sqrt{4-x^2}}{2}\right)^5\right) + C$ 1

10

4. $\int \frac{3x+5}{x^2+11x+10}dx = \int \frac{3x+5}{(x+10)(x+1)} dx = \int \frac{25}{9}\frac{1}{x+10} + \frac{2}{9}\frac{1}{x+1}\, dx$ 1

$= \frac{25}{9}\ln|(x+10)| + \frac{2}{9}\ln|x+1| + C$ 1

2 2

$\frac{3x+5}{(x+10)(x+1)} \overset{2}{=} \frac{A}{x+10} + \frac{B}{x+1}$

$3x+5 = A(x+1) + B(x+10)$

$-25 = A(-9) \Rightarrow A = \frac{25}{9}$ 2

$2 = B(9) \Rightarrow B = \frac{2}{9}$

10

5. Estimate $\int_2^6 e^{1+\arctan(x)}dx$ using the Trapezoidal Rule with n = 8. Write the sum; you do not have to evaluate the sum. 4

$\Delta x = \frac{6-2}{8} = \frac{1}{2}$

$\frac{\frac{1}{2}}{2}\left(f(2) + 2f(2.5) + 2f(3) + 2f(3.5) + 2f(4) + 2f(4.5) + 2f(5) + 2f(5.5) + f(6)\right)$

2 1 1 1 2 2 1

2 —— 2 —— 6

6. Calculate the following; it the integral does not converge, state "does not converge."

8

a. $\int_1^{+\infty} xe^{-x^2}dx = \lim_{b\to+\infty} \int_1^b xe^{-x^2}dx = \lim_{b\to+\infty} -\frac{1}{2}e^{-x^2}\Big|_1^b = \lim_{b\to+\infty} \frac{1}{2}e^{-1} - \frac{1}{2}e^{-b^2}$ 2

2 2

$= \boxed{\frac{1}{2}e^{-1}}$ 2

Converges 2

8

b. $\int_1^3 \frac{4x}{\sqrt{9-x^2}}dx = \lim_{b\to 3^-} \int_1^b \frac{4x}{\sqrt{9-x^2}}dx = \lim_{b\to 3^-} -4\sqrt{9-x^2}\Big|_1^b$ 2 2

$= \lim_{b\to 3^-} -4\sqrt{9-b^2} + 4\sqrt{8} = \boxed{4\sqrt{8}}$ 2 2

1 —— b 3

converges 2

7. Tell why the following converge or diverge:

7

a. $\int_1^{+\infty} \frac{x^2+7}{(x^8+24)^{0.5}}dx$

3 $\frac{x^2+7}{(x^8+24)^{.5}} \le \frac{x^2+7x^2}{(x^8)^{.5}} = \frac{8x^2}{x^4} = \frac{8}{x^2}$

$\int_1^{+\infty} \frac{8\,dx}{x^2}$ converges ($p = 2$) 2

By comparison, $\int_1^{+\infty} \frac{x^2+7}{(x^8+24)^{.5}}dx$ converges 2

7

b. $\int_1^{+\infty} \frac{x^2+7}{(x^4+24)^{0.5}}dx$ 3

$\frac{1}{5} = \frac{x^2}{5x^2} = \frac{x^2}{(x^4+24x^4)^{.5}} \le \frac{x^2+7}{(x^4+24)^{.5}}$ 2

$\int_1^{+\infty} \frac{1}{5}dx$ diverges 2

$\therefore \int_1^{+\infty} \frac{x^2+7}{(x^4+24)^{.5}}dx$ diverges 2

10 8. Calculate $\int \frac{7x}{x^2+4x+13}dx = \int \frac{7x}{(x+2)^2+3^2}dx$ 2

$= \int \frac{7(x+2)}{(x+2)^2+3^2}dx - 14\int \frac{1}{(x+2)^2+3^2}dx$ 1

$= \frac{7}{2}\ln(x^2+4x+13) - \frac{14}{3}\arctan\left(\frac{x+2}{3}\right) + C$

3 3 1

10 9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the integral): $\int \frac{4x+5}{(x^2+6x+9)^3(x^2+8x+20)^2}dx$

2 $\frac{4x+5}{(x+3)^6((x+4)^2+2^2)^2}$ 2

$= \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3} + \frac{D}{(x+3)^4} + \frac{E}{(x+3)^5} + \frac{F}{(x+3)^6}$ 3

$+ \frac{Gx+H}{x^2+8x+20} + \frac{Ix+J}{(x^2+8x+20)^2}$ 3