MAT 162 Test 4 Name Key

Spring 2015 v1

For full credit, show all work.

1. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^5 - 2n + 1}$

12 $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{4n^5-2n+1} \right| = \sum_{n=1}^{n} \frac{1}{4n^5-2n+1} \le \sum_{n=1}^{\infty} \frac{1}{4n^5-2n^5} = \sum_{n=1}^{\infty} \frac{1}{2}\frac{1}{n^5}$ converges by $p=5>1$ series

$\therefore$ By comparison, $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^5-2n+1}$ converges absolutely

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{5n+2}$, $a_n = (-1)^n \frac{n+1}{5n+2}$, $\lim_{n \to +\infty} a_n \neq 0$.

12 $\therefore \sum_{n=1}^{\infty} a_n$ diverges

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{5n+2}$ $\frac{1}{5n+2} \geq 0$

$\to 0$

12

is decreasing,

$\therefore$ By Alt. Series Test $\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{5n+2}$ converges

but not absolutely since $\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+2}}{5n+2} \right| = \sum_{n=1}^{\infty} \frac{1}{5n+2}$ diverges

$\therefore$ the series converges conditionally

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2. Find the radius and interval of convergence for f(x) = $\sum_{n=1}^{\infty} \frac{(x-5)^{2n} 2^{3n}}{n!}$.

$$\lim_{n\to+\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to+\infty}\left|\frac{(x-5)^{2(n+1)} 2^{3(n+1)}}{(n+1)!} \cdot \frac{n!}{(x-5)^{2n} 2^{3n}}\right|$$

12

$$= \lim_{n\to+\infty} \frac{|x-5|^2 2^3}{n+1} = 0$$

$\therefore R = +\infty$ interval of convergence is $(-\infty, +\infty)$

3. Use a power series to estimate $\int_0^{0.1} (\cos(x^3) - 1)dx$ with an error less than 10^{-10}.

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

2

12

$$\cos(x^3) - 1 = \frac{-x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \cdots$$

2

$$\int_0^{0.1} \frac{-x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \cdots = \frac{-x^7}{2!(7)} + \frac{x^{13}}{4!(13)} - \frac{x^{19}}{6!(19)} \cdots \Big|_0^{.1}$$

2 2

$$= \frac{-.1^7}{14} + \frac{.1^{13}}{4!(13)} < 10^{-10} \quad \therefore \text{answer is } -\frac{.1^7}{14}$$

2 2

4. Use the fact that the following series is a telescoping series to calculate $\sum_{n=3}^{\infty} \frac{1}{n^2 + 4n}$ exactly.

$$\frac{1}{n(n+4)} = \frac{1}{4}\left[\frac{1}{n} - \frac{1}{n+4}\right]$$ 4

12

$$S_n = \cancel{\frac{1}{4}\left[\frac{1}{1} - \frac{1}{5}\right] + \frac{1}{4}\left[\frac{1}{2} - \frac{1}{6}\right]} + \frac{1}{4}\left[\frac{1}{3} - \frac{1}{7}\right] + \frac{1}{4}\left[\frac{1}{4} - \frac{1}{8}\right] + \frac{1}{4}\left[\frac{1}{5} - \frac{1}{9}\right]$$

stops

$$+ \frac{1}{4}\left[\frac{1}{6} - \frac{1}{10}\right] + \frac{1}{4}\left[\frac{1}{7} - \frac{1}{11}\right] + \frac{1}{4}\left[\frac{1}{8} - \frac{1}{12}\right] + \frac{1}{4}\left[\frac{1}{9} - \frac{1}{13}\right] + \cdots$$

4

stop

$$\lim S_n = \frac{1}{4}\left[\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right] = \frac{1}{4}\cdot\frac{57}{60} = \frac{57}{240} = \frac{19}{80}$$

4

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5. Calculate $\sum_{n=0}^{\infty} \frac{5^{2n}2^{n+2}}{3^{4n}}$ exactly.

$$= 2^2 \sum_{n=0}^{\infty} \left(\frac{50}{81}\right)^n$$

$$= 4 \left[\frac{1}{1-\frac{50}{81}}\right]$$

12

6. Use the integral test to determine the number of terms in the partial sum for $\sum_{n=1}^{\infty} \frac{2n}{(n^2+1)^6}$ that will estimate the infinite series with an error less than 10^{-6}.

$$f(x) = \frac{2x}{(x^2+1)^6}$$

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$$|R_n(x)| \leq \int_n^{+\infty} \frac{2x}{(x^2+1)^6} = \lim_{b \to +\infty} \frac{(x^2+1)^{-5}}{-5}\Bigg|_n^b = \frac{1}{5}\frac{1}{(x^2+1)^5}\Bigg|_n$$

$$= \frac{1}{5(n^2+1)^5} < 10^{-6}$$

$$= 2 \; 10^5 < (n^2+1)^5$$

$$2^{\frac{1}{5}}(10) < n^2+1$$

$$= \sqrt{2^{\frac{1}{5}}(10) - 1} < n$$

$$3.29 < n$$

$$n = 4 \quad \sum_{n=1}^{4} \frac{2n}{(n^2+1)^6}$$

MAT 162 Test 4 Name Key

Spring 2015 v2

For full credit, show all work.

1. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{7n^5 - 2n + 1}$

12

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{7n^{5} - 2n + 1} \right| = \sum_{n=1}^{\infty} \frac{1}{7n^{5} - 2n + 1} \le \sum_{n=1}^{\infty} \frac{1}{7n^{5} - 2n^{5}} = \sum_{n=1}^{\infty} \frac{1}{5n^{5}}$$

converges $p=5$

$\therefore$ series is absolutely convergent by comparison test to $\sum_{n=1}^{\infty} \frac{1}{5n^{5}}$

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{5n+2n^2}$

12

$$\lim_{n \to +\infty} (-1)^n \frac{n^2+1}{5n+2n^2} \neq 0$$

$\therefore$ $\boxed{\text{series is divergent}}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{5n-2}$

12

converges by alternating series test $\left\{ \begin{array}{l} \frac{1}{5n-2} \geq 0 \\ \text{decreasing} \\ \text{and } \lim \frac{1}{5n-2} = 0 \end{array} \right.$

but $\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+2}}{5n-2} \right| = \sum_{n=1}^{\infty} \frac{1}{5n-2}$ diverges by comparison to $\sum \frac{1}{5n}$

$\therefore$ $\boxed{\text{conditional convergence}}$

2. Find the radius and interval of convergence for $f(x) = \sum_{n=1}^{\infty} \frac{(x-5)^{2n} 2^{3n}}{n}$.

$$\lim_{n \to +\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to+\infty} \left| \frac{(x-5)^{2n+2} \cdot 2^{3n+3}}{n+1} \cdot \frac{n}{(x-5)^{2n} 2^{3n}} \right|$$

$$= \lim_{n\to+\infty} |x-5|^2 2^3 \frac{n}{n+1} = |x-5|^2 8 < 1$$

12

$$|x-5| < \frac{1}{8^{\frac{1}{2}}}$$

$$-\frac{1}{8^{\frac{1}{2}}} < x-5 < \frac{1}{8^{\frac{1}{2}}}$$

$$5-\frac{1}{8^{\frac{1}{2}}} < x < 5+\frac{1}{8^{\frac{1}{2}}}$$

at $x = 5+\frac{1}{8^{\frac{1}{2}}}$ $\sum_{n=1}^{+\infty} \frac{1}{n}$ diverges

at $x = 5-\frac{1}{8^{\frac{1}{2}}}$ $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

$R = \frac{1}{8^{\frac{1}{2}}}$ interval $(5-\frac{1}{8^{\frac{1}{2}}}, 5+\frac{1}{8^{\frac{1}{2}}})$

3. Use a power series to estimate $\int_0^{0.1} (\cos(x^3) + .5x^6 - 1)dx$ with an error less than 10^{-15}.

12

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$\cos x^3 = 1 - \frac{x^6}{2} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \cdots$$

$$\cos x^3 + .5x^6 - 1 = \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \cdots$$

$$\int_0^{.1} \cos(x^3) + .5x^6 - 1 = \frac{x^{13}}{4!\,13} - \frac{x^{19}}{6!\,(19)} + \cdots \Big|_0^{.1}$$

$< 10^{-15}$

$$= \frac{(.1)^{13}}{4!\,(13)}$$

4. Use the fact that the following series is a telescoping series to calculate $\sum_{n=3}^{\infty} \frac{1}{n^2+3n}$ exactly.

12

$$\frac{1}{n^2+3n} = \frac{1}{3}\left[\frac{1}{n} - \frac{1}{n+3}\right]$$

$$S_n = \frac{1}{3}\left[\frac{1}{1} - \frac{1}{4}\right] + \frac{1}{3}\left[\frac{1}{2} - \frac{1}{5}\right] + \frac{1}{3}\left[\frac{1}{3} - \frac{1}{6}\right] + \frac{1}{3}\left[\frac{1}{4} - \frac{1}{7}\right] + \frac{1}{3}\left[\frac{1}{5} - \frac{1}{8}\right]$$

$$+ \frac{1}{3}\left[\frac{1}{6} - \frac{1}{9}\right] + \frac{1}{3}\left[\frac{1}{7} - \frac{1}{10}\right] + \cdots$$

$$\lim_{n\to+\infty} S_n = \frac{1}{3}\left[1 + \frac{1}{2} + \frac{1}{3}\right] = \frac{1}{3}\left[\frac{6}{6} + \frac{3}{6} + \frac{2}{6}\right] = \frac{11}{3(6)} = \frac{11}{18}$$

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$$= \frac{1}{3}\left[\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right]$$

$$= \frac{1}{3}\left[\frac{20+15+12}{60}\right]$$

$$= \frac{47}{180}$$

5. Calculate $\sum_{n=0}^{\infty} \frac{5^{2n} 2^{n+4}}{3^{4n}}$ exactly.

$$2^4 \sum_{n=0}^{\infty} \frac{(5^2)^n \cdot 2^n}{81^n} = 2^4 \sum \left(\frac{50}{81}\right)^n$$

$$= 16 \left[\frac{1}{1-\frac{50}{81}}\right]$$

$$= 16 \left[\frac{81}{31}\right]$$

12

6. Use the integral test to determine the number of terms in the partial sum for $\sum_{n=1}^{\infty} \frac{2n}{(n^2+1)^8}$ that will estimate the infinite series with an error less than 10^{-6}.

$$R_n \le \int_n^{+\infty} \frac{2x}{(x^2+1)^8}\,dx = \lim_{b \to +\infty} \frac{(x^2+1)^{-7}}{-7}\Big|_n^b = \frac{1}{(n^2+1)^7(7)} < 10^{-6}$$

16

$$\frac{10^6}{7} \le (n^2+1)^7$$

$$\left(\frac{10^7}{70}\right)^{\frac{1}{7}} \le n^2+1$$

$$\frac{10}{70^{1/7}} \le n^2+1$$

$$2.10956 \left(\frac{10}{70^{\frac{1}{7}}} - 1\right)^{\frac{1}{2}} \le n$$

$$3 \le n$$

$$\sum_{n=1}^{3} \frac{2n}{(n^2+1)^8}$$