v.1

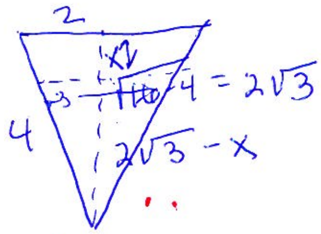
Spring, 2015 Name: Key

MAT 162001 Test 2 Version 1 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book. Each problem is worth 20 points. Problem 6 is 20 points extra credit.

(20)

1. A large trough ▼ at one end is an equilateral triangle with the horizontal top 4 feet wide .. The trough is filled with corn syrup of density 90 lbs per cubic foot. What is the force on the triangular end of the trough?



$\frac{2\sqrt{3} - x}{2\sqrt{3}} = \frac{w}{4}$

$w = 4(1 - \frac{1}{2\sqrt{3}}x)$

(18)

Area = $4(1 - \frac{1}{2\sqrt{3}}x)\Delta x$

depth = x

$\int_0^{2\sqrt{3}} 90x(4 - \frac{4}{2\sqrt{3}}x)\,dx = \int_0^{2\sqrt{3}} 90(4x - \frac{4}{2\sqrt{3}}x^2)\,dx$

$= 90(2x^2 - \frac{4}{2\sqrt{3}}\frac{1}{3}x^3)\Big|_0^{2\sqrt{3}} = 90(2(12) - \frac{4}{2\cdot 3\sqrt{3}}2\sqrt{3}(12))$

$= 90(24 - 4) = 90(\cancel{20}\ 8) = \boxed{\cancel{1800}}\ 720$

(20)

2. Suppose the average waiting time for a customer's call to be answered by a company representative (modeled by an exponentially decreasing probability density function) is 6 minutes. Find the median waiting time. Find the probability that is takes more than 6 minutes for the call to be answered.

(2) $f(x) = \frac{1}{6}e^{-\frac{1}{6}x}$ if $x \geq 0$ otherwise 0

Find M so that $.5 = \int_0^M \frac{1}{6}e^{-\frac{1}{6}x}\,dx = -e^{-\frac{1}{6}x}\Big|_0^M = 1 - e^{-\frac{1}{6}M}$

(2) 2 1

$e^{-\frac{1}{6}M} = .5$ 1

$-\frac{1}{6}M = \ln .5$ 1

$\boxed{M = -6\ln .5}$ 1 $= 6\ln 2$

1

$P(x>6) = 2\int_6^{+\infty} \frac{1}{6}e^{-\frac{1}{6}x}\,dx = \lim_{b\to+\infty} -e^{-\frac{1}{6}x}\Big|_6^b = \boxed{e^{-1}}$

2 2

1 2

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$x^3 - 9x \ge \frac{1}{2}x^3$

$\frac{1}{2}x^3 \ge 9x$

$x^2 \ge 18$

$x \ge \sqrt{18}$

6. Let $f(x) = k/(x^3-9x)$ for $x \ge 4$ (for $x < 4$, $f(x) = 0$).

(6)

a. Without evaluating the improper integral of f(x), tell why we know there is a value k such that f(x) is a probability density function.

$\frac{k}{x^3-9x} \le \frac{k}{\frac{1}{2}x^3}$ for $x \ge 18$ 2

$\int_4^{+\infty} \frac{k}{\frac{1}{2}x^3}\,dx$ converges $\Rightarrow \int_4^{+\infty} \frac{k}{x^3-9x}\,dx$ to converge 2

2

By the comparison Test.

(7)

b. Find the value of k in order that f(x) is a probability density function.

$\frac{1}{k} = \int_4^{+\infty} \frac{1}{x^3-9x}\,dx = \int_4^{+\infty} \frac{1}{x(x-3)(x+3)}\,dx = \lim_{b\to+\infty}\left(-\frac{1}{9}\ln x + \frac{1}{18}\ln|x-3| + \frac{1}{18}\ln|x+3|\right)\Big|_4^b$

$\frac{1}{x(x-3)(x+3)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+3}$

$1 = A(x-3)(x+3) + Bx(x+3) + Cx(x-3)$

let $x=0$ $\quad 1 = A(-9) \Rightarrow A = -\frac{1}{9}$

$x=3$ $\quad 1 = B\,3(6) \Rightarrow B = \frac{1}{18}$

$x=-3$ $\quad 1 = C(-3)(-6) \Rightarrow C = \frac{1}{18}$

$= \lim_{b\to+\infty} -\frac{1}{9}\ln b + \frac{1}{18}\ln|b-3| + \frac{1}{18}\ln(b+3) + \frac{1}{9}\ln 4 - \frac{1}{18}\ln 7$ 2

$= \lim_{b\to+\infty} \frac{1}{18}\ln\frac{(b-3)(b+3)}{b^2}$ 0 $+ \frac{1}{9}\ln 4 - \frac{1}{18}\ln 7$

$\therefore$ $\boxed{k = \frac{1}{\frac{1}{9}\ln 4 - \frac{1}{18}\ln 7}}$

(7)

c. Find the mean of the probability density function f(x).

$\mu = k\int_4^{+\infty} \frac{1}{(x-3)(x+3)}\,dx = k\lim_{b\to+\infty} \frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3|\Big|_4^b$

$\frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$

$1 = A(x+3) + B(x-3)$

$x=3$ $\quad 1 = A6 \Rightarrow A = \frac{1}{6}$

$x=-3$ $\quad 1 = B(-6) \Rightarrow B = -\frac{1}{6}$

$= k\left(\frac{1}{6}\ln 7 - \cancel{\ln 1}\right)$ 0

$= \frac{\left(\frac{1}{6}\ln 7\right)}{\left(\frac{1}{9}\ln 4 - \frac{1}{18}\ln 7\right)}$ 1

2

=

v.1

3. Find the length of the curve $y = 3+4x$, $2 \le x \le 5$.

20

$$2\int_2^5 \sqrt{1+(f'(x))^2}\,dx = \int_2^5 \sqrt{1+4^2} = \sqrt{17}\,x\Big|_2^5 = \sqrt{17}(5-2)$$

2 2 2 3 3 4

$$= \boxed{\sqrt{17}\ 3}$$

12.36931688

4. Find the area of the surface obtained by rotating the curve $y = 3 + 4x$, $2 \le x \le 5$, about the x-axis.

20

$$2\int_2^5 2\pi f(x)\sqrt{1+|f'(x)|^2}\,dx = \int_2^5 2\pi(3+4x)\sqrt{17}\,dx \quad 3$$

2 2

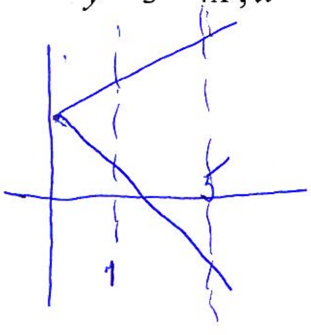
$$= 2\pi\sqrt{17}\,(3x+2x^2)\Big|_2^5 \quad 3$$

$$= 2\pi\sqrt{17}\,(15+50-6-8) \quad 4$$

$$= 2\pi\sqrt{17}\,(51) = \boxed{102\pi\sqrt{17}} = 1321.218$$

5. Find the centroid (center of mass) of the region bounded by the curves: $y = 3 + 4x$, $y = 3 - 4x$, $x = 1$ and $x = 5$.

20



(4) $\text{Mass} = \int_1^5 \rho(3+4x-(3-4x))\,dx = \int_1^5 \rho(8x)\,dx = \rho 4x^2\Big|_1^5 = \rho(100-4) = 96\rho$

(4) $M_x = \int_1^5 \rho\left(\frac{3+4x+3-4x}{2}\right)(3+4x-(3-4x))\,dx = \int_1^5 \frac{\rho}{2}\left[(3+4x)^2-(3-4x)^2\right]dx = \int_1^5 \frac{\rho}{2}[48x] = \int_1^5 \rho(24x) = \rho 12x^2\Big|_1^5 = \rho 12(25-1) = \rho 12(24) = 2\rho 144 = 288\rho$

(4) $M_y = \int_1^5 \rho x(3+4x-(3-4x))\,dx = \int_1^5 \rho x\, 8x\,dx = \frac{8}{3}x^3\Big|_1^5 = \frac{8}{3}\rho(125-1) = \frac{8}{3}\rho(124) = 4\cancel{1(8\rho)}$ $= \cancel{328\rho}$ $\frac{992}{3}\rho$

(4) $\bar{x} = \frac{M_y}{M} = \frac{\cancel{328\rho}\ 992}{3\ 96\rho} = \cancel{3.416}\ 3.448$

(4) $\bar{y} = \frac{M_x}{M} = \frac{288\rho}{96\rho} = \boxed{3}$

V. 2

Spring, 2015 Name: Key

MAT 162001 Test 2 Version 2 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book. Each problem is worth 20 points. Problem 6 is 20 points extra credit.

1. A large inverted trough Δ at one end is an equilateral triangle with the horizontal bottom 4 feet wide .. The trough is filled with corn syrup of density 90 lbs per cubic foot. What is the force on the triangular end of the trough?

x = depth

$$\frac{x}{2\sqrt{3}} = \frac{w}{4}$$

$$w = 4\frac{x}{2\sqrt{3}} = \frac{2x}{\sqrt{3}}$$

$$\text{Area} = \frac{2x}{\sqrt{3}}\Delta x$$

$$\int_0^{2\sqrt{3}} 90x\left(\frac{2}{\sqrt{3}}x\right)dx = \int_0^{2\sqrt{3}} \frac{180}{\sqrt{3}}x^2\,dx = \frac{180}{\sqrt{3}}\frac{x^3}{3}\Big|_0^{2\sqrt{3}}$$

$$= \frac{180}{\sqrt{3}}\frac{1}{3}2^3\, 3\sqrt{3} = 180(8) = \boxed{1440}$$

2. Suppose the average waiting time for a customer's call to be answered by a company representative (modeled by an exponentially decreasing probability density function) is 8 minutes. Find the median waiting time. Find the probability that is takes more than 9 minutes for the call to be answered.

$f(x) = \frac{1}{8}e^{-\frac{1}{8}x}$ if $x \geq 0$ otherwise 0

Find M so that $.5 = \int_0^M \frac{1}{8}e^{-\frac{1}{8}x}dx = -e^{-\frac{1}{8}x}\Big|_0^M = 1 - e^{-\frac{1}{8}M}$

$$e^{-\frac{1}{8}M} = .5$$

$$-\frac{1}{8}M = \ln .5$$

$$M = -8\ln .5 = 8\ln 2 \approx 5.545177444$$

$$P(x>9) = \int_9^{+\infty}\frac{1}{8}e^{-\frac{1}{8}x}dx = \lim_{b\to+\infty} -e^{-\frac{1}{8}x}\Big|_9^b = \boxed{e^{-\frac{9}{8}}} \approx .3246524674$$

n 2

(20) 3. Find the length of the curve $y = 4+3x$, $2 \le x \le 5$.

$$L = \int_2^5 \sqrt{1+3^2}\,dx = \int_2^5 \sqrt{10}\,dx = \sqrt{10}\,x\Big|_2^5 = \sqrt{10}(5-2)$$

$$= \boxed{\sqrt{10}(3)} \quad 4$$

2 2 2 3 3

4. Find the area of the surface obtained by rotating the curve $y = 4+3x$, $2 \le x \le 5$, about the x-axis.

20

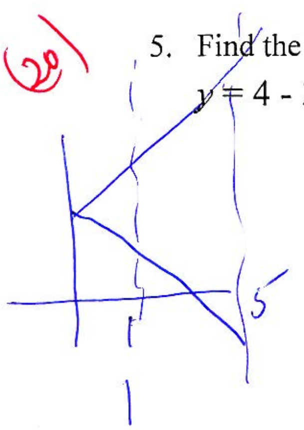
$$\int_2^5 2\pi(4+3x)\sqrt{10}\,dx = 2\pi\sqrt{10}\left(4x + \frac{3}{2}x^2\right)\Big|_2^5 \quad 3$$

$$= 2\pi\sqrt{10}\left(20 + \frac{3(25)}{2} - (8+6)\right) \quad 4$$

$$= 2\pi\sqrt{10}\left(6 + \frac{75}{2}\right) = 2\pi\sqrt{10}\left(\frac{87}{2}\right) = 87\pi\sqrt{10}$$

$$= 864.309$$

(20) 5. Find the centroid (center of mass) of the region bounded by the curves: $y = 4+3x$, $y = 4-3x$, $x = 1$ and $x = 5$.



(5) $$\text{Mass} = \int_1^5 \rho\left(4+3x-(4-3x)\right)dx = \int_1^5 \rho\,6x\,dx$$

$$= \rho\,6\,\frac{1}{2}x^2\Big|_1^5 = 3\rho(25-1) = 3\rho(24) = 72\rho$$

(4) $$M_x = \int_1^5 \frac{\rho}{2}\left((4+3x)^2-(4-3x)^2\right)dx = \int_1^5 \frac{\rho}{2}(48x)\,dx$$

$$= \rho\frac{24x^2}{2}\Big|_1^5 = 12\rho(25-1) = 12\rho(24) = 2\rho 144 = 288\rho$$

(4) $$M_y = \int_1^5 \rho x\left(4+3x-(4-3x)\right)dx = \int_1^5 \rho x\,6x\,dx = 2\rho x^3\Big|_1^5$$

$$= 2\rho(125-1) = 2\rho(124) = 248\rho$$

(4) $$\bar{x} = \frac{M_y}{M} = \frac{248\rho}{72\rho} = \boxed{3.4}$$

(4) $$\bar{y} = \frac{M_x}{M} = \frac{288\rho}{72\rho} = \boxed{4}$$

V. 2

$x^3 - 4x \geq \frac{1}{2}x^3$

$\frac{1}{2}x^3 \geq 4x$

$x^2 \geq 8$

$x > \sqrt{8}$

(6) 6. Let f(x) = k/(x^3-4x) for x ≥ 5 (for x < 5, f(x) = 0).

a. Without evaluating the improper integral of f(x), tell why we know there is a value k such that f(x) is a probability density function.

2

$\frac{k}{x^3-4x} \leq \frac{k}{\frac{1}{2}x^3}$ for $x \geq 8$

$\int_5^{+\infty} \frac{8}{\frac{1}{2}x^3}\,dx$ converges $\Rightarrow \int_5^{+\infty} \frac{k}{x^3-4x}\,dx$ converges

2 2

(7)

b. Find the value of k in order that f(x) is a probability density function.

$\frac{1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$

$1 = A(x-2)(x+2) + Bx(x+2) + C(x)(x-2)$

$x=0$ $1 = A(-4) \Rightarrow A = -\frac{1}{4}$

$x=2$ $1 = B(2)(4) \Rightarrow B = \frac{1}{8}$

$x=-2$ $1 = C(-2)(-4) \Rightarrow C = \frac{1}{8}$

$\frac{1}{k} = \int_5^{+\infty} \frac{1}{x(x-2)(x+2)}\,dx = \lim_{b \to +\infty} \left(-\frac{1}{4}\ln|x| + \frac{1}{8}\ln|x-2| + \frac{1}{8}\ln|x+2|\right)\Big|_5^b$

$= \lim_{b \to +\infty} \frac{1}{8}\ln\left(\frac{(x-2)(x+2)}{x^2}\right)\Big|_5^b = -\frac{1}{8}\ln\left|\frac{3(7)}{25}\right|$

$= \frac{1}{8}\ln\frac{25}{21} = .0217941734$

(7) $k = \frac{1}{\frac{1}{8}\ln\frac{25}{21}} = 45.88382326$

c. Find the mean of the probability density function.

$\mu = \int_5^{+\infty} \frac{k}{(x-2)(x+2)}\,dx = k \lim_{b \to +\infty} \left(\frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2|\right)\Big|_5^b$

$\frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$

$1 = A(x+2) + B(x-2)$

$x = 2$ $1 = A(4) \Rightarrow A = \frac{1}{4}$

$x = -2$ $1 = B(-4) =$

$B = -\frac{1}{4}$

$= k\left(\frac{1}{4}\ln 7 - \frac{1}{4}\ln 3\right)$

$\boxed{= \frac{k}{4}\ln\frac{7}{3}}$ $= 9.719314378$