MAT 162 Final Exam Name _____
Fall 2014 Version 2

For full credit, show all work.

I. Calculate the following"

a. $\int x\sin^2(x)\ dx$

b. $\int x^3(1+x^2)^{1/2}dx$

II. Tell whether $\int_1^{+\infty}\frac{1+\cos(\sin(5x^3))}{x^3+17}dx$ converges or diverges, and why.

III. Use the Simpson's rule with n = 4 to estimate $\int_4^7 \frac{x}{1+x^6}\,dx$ .

IV. Find the length of the graph of the curve $y = 9 + 8x^{1.5}$, $0 \le x \le 3$ .

V. Find the centroid of the region bounded by the curves $y = x^2$ and $20 = x^2 + y^2$ . Set up the integrals; you do not have to solve them.

VI. Find k so that $f(x) = \frac{k}{x^2+10x}$ if $x \geq 4$ and $f(x) = 0$ if $x < 4$, is a probability density function.

VII. Solve completely:

(a) $\frac{dy}{dx} = \frac{1+y}{1+8x^2}$ , $y(0) = 1$.

(b) $\frac{dy}{dx} + 6y = 5e^{3x}$

(c) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 0$.

VIII. Use Euler's Method and a stepsize of h = 0.1 to estimate y(.2) where $\frac{dy}{dx} = x(1+3y)^2$, y(0) = 2.

IX. A 1000 liter tank is initially filled with brine that contains dissolved salt. A salt solution of .004 kg/l enters the tank at a rate of 50 l/minute; the tank is continuously mixed and a solution drains from the tank at a rate of 70 l/minute. In 10 minutes there is exactly 1 kg of salt in the tank. How much salt was in the tank in the beginning?

X. Find the foci and vertices and sketch the graph of $x^2 - 6x + 4y^2 + 16y = -21$.

XI. Convert $r = 2/(1+.5\sin(\theta))$ into rectangular coordinates and sketch the graph. Find the slope of the tangent line at $\theta = \frac{\pi}{2}$.

XII. For $x = t - 2t^2$ and $y = 24t^4 - 6t^2$, $-1 < t < 3$

(a) Find the points where the parametric system has a vertical tangent line.

(b) Find the points where there are horizontal tangent lines.

(c) Find where x is increasing.

(d) Find where y is increasing.

(e) Sketch the graph of the system on an x-y coordinate system.

XIII. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^3 + 5}$

(b) $\sum_{n=1}^{\infty} (-1)^n 9^n e^{-2^n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n^4)}{n^3}$

XIV. Find the radius and interval of convergence for f(x) = $\sum_{n=1}^{\infty}(8x+12)^n 4^{-2n}$ .

XV. Use a power series to estimate $\int_0^{0.1}\frac{\cos(x^6)-1}{4x^3}dx$ with an error less than $10^{-25}$.