MAT 162 Final Exam Name _____
Fall 2014 Version 1

For full credit, show all work.

I. Calculate the following"

a. $\int x\cos^2(x)\ dx$

b. $\int x^3(1-x^2)^{1/2}dx$

II. Tell whether $\int_1^{+\infty}\frac{1+\cos(5x)}{x^3+17}dx$ converges or diverges, and why.

III. Use the Simpson's rule with n = 6 to estimate $\int_4^7 \frac{x}{1+x^6}\,dx$.

IV. Find the length of the graph of the curve $y = 10 + 4x^{1.5}$, $0 \le x \le 3$.

V. Find the centroid of the region in the first and second quadrants bounded by the curves $y = (1/7)(x + 50)$,and $100 = x^2 + y^2$. Set up the integrals you do not have to solve them.

VI. Find k so that $f(x) = \frac{k}{x^2 + 10x}$ if $x \geq 4$ and $f(x) = 0$ if $x < 4$, is a probability density function.

VII. Solve completely:

(a) $\frac{dy}{dx} = \frac{1+y^2}{1+8x}$, $y(0) = 1$.

(b) $\frac{dy}{dx} + 8y = 5e^{3x}$

(c) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 0$.

VIII. Use Euler's Method and a stepsize of h = 0.1 to estimate y(.2) where $\frac{dy}{dx} = x+(1+3y)^2$, y(0) = 2.

IX. A 1000 liter tank is initially filled with brine that contains dissolved salt. A salt solution of .004 kg/l enters the tank at a rate of 50 l/minute; the tank is continuously mixed and a solution drains from the tank at a rate of 70 l/minute. In 20 minutes there is exactly 1 kg of salt in the tank. How much salt was in the tank in the beginning?

X. Find the foci and vertices and sketch the graph of $2x^2 + 4x + y^2 + 16y = 34$.

XI. Convert $r = 6\cos(\theta) + 4\sin(\theta)$ into rectangular coordinates and sketch the graph. Find the slope of the tangent line at $\theta = \frac{\pi}{2}$.

XII. For $x = 4 - t^2$ and $y = 3t^4 - 3t^2$, $1 < t < 3$

(a) Find the points where the parametric system has a vertical tangent line.

(b) Find the points where there are horizontal tangent lines.

(c) Find where x is increasing.

(d) Find where y is increasing.

(e) Sketch the graph of the system on an x-y coordinate system.

XIII. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^3 + 5}$

(b) $\sum_{n=1}^{\infty} (-1)^n 9^n e^{-3^n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)}{n^3}$

XIV. Find the radius and interval of convergence for f(x) = $\sum_{n=1}^{\infty}(6x+12)^n 4^{-2n}$.

XV. Use a power series to estimate $\int_0^{0.1}\frac{\cos(x^5)-1}{4x^3}dx$ with an error less than 10^{-25}.