MAT 162 Test 4 Name Key

Spring 2014

For full credit, show all work.

1. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{2+\sin(n^2)}{n}$

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$\frac{1}{n} \leq \frac{2+\sin(n^2)}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges $\Rightarrow \sum_{n=1}^{\infty} \frac{2+\sin(n^2)}{n}$ diverges by comparison Test

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n^3+1}{\sqrt{n^7}+6}$

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conditional convergence

$a_n = f(n)$ where $f(x) = \frac{x^3+1}{x^{7/2}+6}$, $f(x) > 0$ for $x \geq 0$

$\lim_{x\to+\infty} f(x) = 0$

$f'(x) = \frac{(x^{7/2}+6)(3x^2) - (x^3+1)\frac{7}{2}x^{5/2}}{(x^{7/2}+6)^2} = \frac{-\frac{1}{2}x^{11/2}+18x^2-\frac{7}{2}x^{5/2}}{(x^{7/2}+6)^2}$

$= \frac{x^2(-\frac{1}{2}x^{7/2}-\frac{7}{2}x^{\frac{1}{2}}+18)}{(x^{7/2}+6)^2} < 0$

$\therefore$ f decreasing eventually

Alt. Series Test implies convergence

Abs convergence no $\sum_{n=1} \frac{n^3+1}{n^{7/2}+6}$ diverges by comparison to $\sum \frac{1}{n^{\frac{1}{2}}}$ compare to $\frac{1}{n^{\frac{1}{2}}}$ $\frac{n^{7/2}+n^{\frac{1}{2}}}{n^{7/2}+6} \to 1$

(c) $\sum_{n=1}^{\infty} (\frac{-5}{2n+3})^n$

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$\lim_{n\to+\infty} |a_n|^{\frac{1}{n}} = \lim_{n\to+\infty} \frac{5}{2n+3} = 0$

$\therefore$ absolute convergence

2. Find the radius and interval of convergence for $f(x) = \sum_{n=1}^{\infty} (x-4)^n 5^{-n} (1/n)$.

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$$\lim_{n\to+\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to+\infty} \left|\frac{(x-4)^{n+1}}{5^{n+1}(n+1)} \cdot \frac{5^{n}\, n}{(x-4)^n}\right|$$

$$= \lim_{n\to+\infty} \frac{|x-4|}{5} \frac{n}{n+1} = \frac{|x-4|}{5} < 1$$

$R = 5$,

$-5 < x-4 < 5$

$-1 < x < 9$

at $x = 9$, $\sum \frac{1}{n}$ diverges

at $x = -1$ $\sum_n (-1)^n \frac{1}{n}$ converges

interval of convergence is $[-1, 9)$

3. Use a power series to estimate $\int_0^{0.1} e^{-x^2}\,dx$ with an error less than 10^{-5}.

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$$e^{-x^2} = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{k!}$$

$$\int e^{-x^2} = C + \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{k!(2k+1)}$$

$$\int_0^{.1} e^{-x^2}dx = \sum_{k=0}^{\infty} (-1)^k \frac{(.1)^{2k+1}}{k!(2k+1)} = \frac{.1}{1} - \frac{(.1)^3}{1!(3)} + \underline{\frac{(.1)^5}{2!(5)}}$$

10^{-5}

$$\boxed{.1 - \frac{(.1)^3}{3}}$$

4. Use the fact that the following series is a telescoping series to calculate $\sum_{n=4}^{\infty} \frac{1}{n^2 - 3n}$ exactly.

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$$\frac{1}{n(n-3)} = \frac{A}{n} + \frac{B}{n-3} = \frac{1}{3}\left[\frac{1}{n-3} - \frac{1}{n}\right]$$

$1 = A(n-3) + Bn$

$n = 0$ $\quad 1 = A(-3) \Rightarrow A = -3$

$n = 3$ $\quad 1 = B\,3 \Rightarrow B = \frac{1}{3}$

$$\sum_{n=4}^{\infty} \frac{1}{3}\left[\frac{1}{n-3} - \frac{1}{n}\right] = \frac{1}{3}\left[\frac{1}{1} - \cancel{\frac{1}{4}}\right] + \frac{1}{3}\left[\frac{1}{2} - \cancel{\frac{1}{5}}\right] + \frac{1}{3}\left[\frac{1}{3} - \cancel{\frac{1}{6}}\right] + \frac{1}{3}\left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right] + \frac{1}{3}\left[\cancel{\frac{1}{5}} - \frac{1}{8}\right] + \frac{1}{3}\left[\cancel{\frac{1}{6}} - \frac{1}{9}\right] + \frac{1}{3}\left[\cancel{\frac{1}{7}} - \frac{1}{10}\right]$$

$$\sum_{n=4}^{\infty} \frac{1}{3}\left[\frac{1}{n-3} - \frac{1}{n}\right] = \frac{1}{3}\left[1 + \frac{1}{2} + \frac{1}{3}\right]$$

5. Calculate $\sum_{n=1}^{\infty} \frac{2^{n-2}7^{n+2}}{3^{3n}}$ exactly. $= 2^{-2} 7^{2} \sum_{n=1} \frac{14^n}{27^n}$

$= \boxed{\frac{49}{4} \frac{\frac{14}{27}}{1-\frac{14}{27}}} = \frac{49}{4} \frac{\frac{14}{27}}{\frac{13}{27}}$

$= \frac{49}{4} \frac{14}{13}$

$= \frac{(49)(7)}{2(13)}$

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6. Use the integral test to determine the number of terms in the partial sum for $\sum_{n=1}^{\infty} \frac{1}{n^5}$ that will estimate the infinite series with an error less than .005

Find n so that $\int_n^{+\infty} \frac{1}{x^5}\,dx < .005$

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$\lim_{a\to+\infty} \int_n^a \frac{1}{x^5}\,dx = \lim_{a\to+\infty} -\frac{1}{4x^4}\Big|_n^a = \frac{1}{4n^4} < .005$

$\frac{1}{4(.005)} < n^4$

$\frac{1}{.02} < n^4$

$50 < n^4$

$3 \leq n$

$\boxed{\frac{1}{1^5} + \frac{1}{2^5} + \frac{1}{3^5}}$

three terms