Test 3 Spring, 2014 Name: Key

MAT 162 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book.

24
-8
12

1. For the parametric curve, $x = t^3 - 12t$, $y = 9 - t^3$, $-3 \le t \le 3$:

(a) Calculate the following: $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$.

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$\frac{dx}{dt} = 3t^2 - 12$

$\frac{dy}{dt} = -3t^2$

$\frac{dy}{dx} = \frac{-3t^2}{3t^2-12} = \frac{-t^2}{t^2-4}$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{-t^2}{t^2-4}\right)}{3t^2-12} = \frac{(t^2-4)(-2t) - t^2(2t)}{(t^2-4)^2(3t^2-12)}$

(b) Find when x is increasing and decreasing.

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$\frac{dx}{dt} = 3(t^2-4) = 3(t-2)(t+2)$ x inc when $t<-2$ and $t>2$

$\frac{dx}{dt} < 0$ when $-1 < t < 1$ x dec when $-2<t<2$

(c) Tell when y is increasing and decreasing.

6

$\frac{dy}{dt} = -3t^2$ y always decreasing

(d) Find the xy coordinates where there is a horizontal tangent line.

$0 = \frac{dy}{dt} = -3t^2 \Rightarrow t=0$ $x=0$ $y=9$

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(e) Are there any vertical tangent lines? If so, what are the xy coordinates where there is a vertical tangent line?

$0 = \frac{dx}{dt} = 3(t-2)(t+2)$ $t=-2$ ~~(11, 10)~~ (16, 17)

6 $t=2$ (-11, 8) (-16, 1)

(f) Sketch the graph of the parametric curve in the xy plane for $-3 \le t \le 3$.

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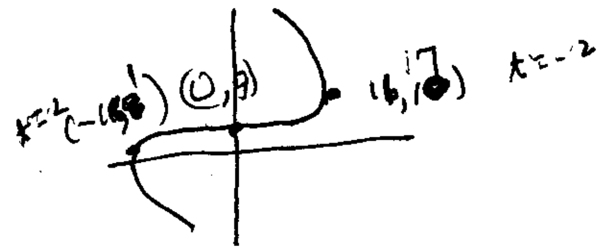
t		
-3	9	36
-2	16	17
0	0	9
2	-16	1
3	-9	-18



(g) Write down, but do not evaluate the integral of the length of the curve.

6 $L = \int_{-3}^{3} \sqrt{(3t^2-12)^2 + (3t^2)^2}\, dt$

2. (a) Sketch the polar graph of r = 2 - cos(θ) from θ = 0 to 2π.. Identify in polar coordinates where the graph crosses the x-axis and y-axis.

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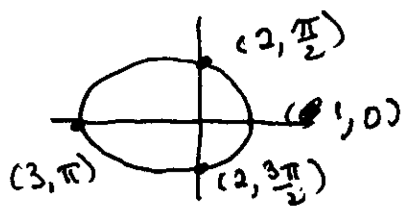

(b) Find the area between the curve and the origin in the first quadrant.

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$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} r^2\, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} (2-\cos\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(4 - 4\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta = \frac{1}{2}\left[\frac{9}{2}\theta - 4\sin\theta + \frac{\sin 2\theta}{2}\right]\Bigg|_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\frac{9}{2}\cdot\frac{\pi}{2} - 4 + 0 - \frac{9}{2}(0) - 4(0) - 0\right] = \frac{1}{2}\left[\frac{9\pi}{4} - 4\right] = 1.534291735$$

(c) Write down, but do not evaluate the integral of the length of this curve.

6 $L = \int_0^{2\pi} \sqrt{(2-\cos\theta)^2 + (\sin\theta)^2}\, d\theta$

III. For $16x^2 - 25y^2 - 32x - 100y = 484$,

(a) Identify the conic section.

$16(x^2 - 2x) - 25(y^2 + 4y) = 484$

6 $16(x^2 - 2x + 1) - 25(y^2 + 4y + 4) = 484 + 16 - 100 = 400$

$\dfrac{(x-1)^2}{5^2} - \dfrac{(y+2)^2}{4^2} = 1$ hyperbola

(b) Find the center.

6 Center $(1, -2)$

(c) Find the vertices.

6 vertices $(1 \pm 5, -2)$ — $\begin{cases}(6, -2)\\(-4, -2)\end{cases}$

(d) Find the foci.

$c^2 = a^2 + b^2 = 25 + 16 = 41 \Rightarrow c = \sqrt{41}$

6 $(1 \pm \sqrt{41}, -2)$

(e) Sketch the curve in the xy plane.

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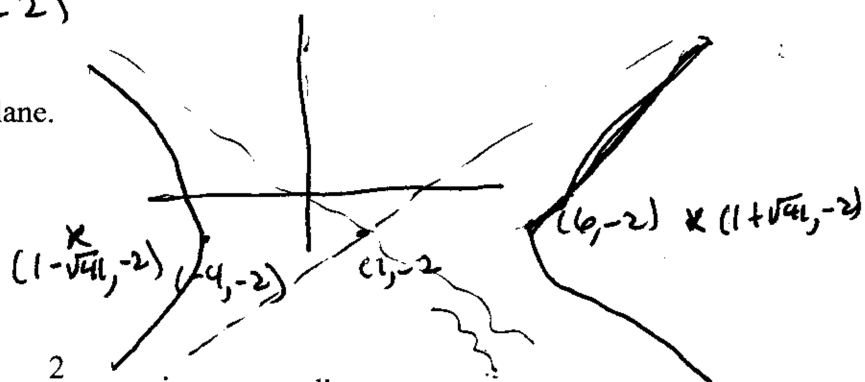

IV. Rewrite the polar equation $r = \dfrac{2}{3+\sin(\theta)}$ using xy coordinates.

~~3r + r sin~~ $r = \dfrac{\frac{2}{3}}{1 + \frac{1}{3}\sin\theta}$

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$r = \frac{2}{3} - \frac{1}{3} r \sin\theta$

$x^2 + y^2 = \left(\frac{2}{3} - \frac{1}{3}y\right)^2$