Test 2 Spring, 2014 Name:______________________

MAT 162 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. The test is closed book.

1. Use Euler's Method to approximate y(2.2) given $\frac{dy}{dx} = 4x - 5y$, and y(2) = 1. Use a stepsize of 0.1 .

12½

x	y	f(x,y) (4x-5y)	.1 f(x,y)
2	1	3	.3
2.1	1.3	1.9	.19
2.2	1.49		

11½

8.4

-6.5

2. Find y(x), the solution to $\frac{dy}{dx} = (y^2+1)(x^2+1)$, y(0) = π/4 .

12½

$\frac{1}{y^2+1}\,dy = (x^2+1)\,dx$ 2 2

$\arctan y = \frac{1}{3}x^3 + x + C$ 2 2

$\arctan \frac{\pi}{4} = C$ 2

$\arctan y = \frac{1}{3}x^3 + x + \arctan \frac{\pi}{4}$ 1

$\boxed{y = \tan\left(\frac{1}{3}x^3 + x + \arctan \frac{\pi}{4}\right)}$ 1½

3. Find y(x), the solution to $\frac{dy}{dx} = e^x + y$, y(0) = 1 .

12½

$\frac{dy}{dx} - y = e^x$ 2

$e^{-x}\frac{dy}{dx} - e^{-x}y = e^x \cdot e^{-x}$

$\frac{d}{dx}(e^{-x}y) = 1$ 2

$e^{-x}y = x + C$ 2

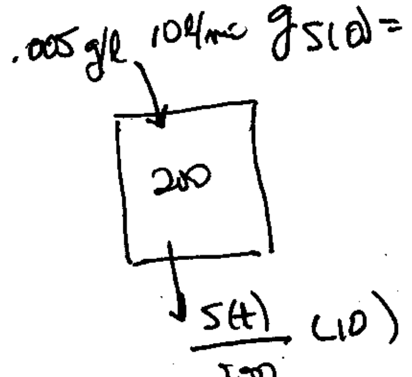
$y = xe^x + Ce^x$ 2

$1 = Ce^0 \Rightarrow C = 1$ 2

$\boxed{y = xe^x + e^x}$ 2

½

4. A tank is filled with 200 liters of contaminated water containing 2 g of toxins. Water containing .005 g of toxin per liter is pumped in at a rate of 10 l/min., mixes instantaneously, and then is pumped out at the same rate. Find y(t) the number of grams of the toxin in the tank t minutes after the rinse begins. Then find the time at which there is 1 ~~kg~~ of toxin present.



$\frac{ds}{dt}$ = rate in − rate out

$\frac{ds}{dt} = .05\ g/\text{min} - \frac{1}{20}S(t)$, $S(0) = 2$

$\frac{ds}{dt} + \frac{1}{20}S(t) = .05$

$\frac{d}{dt}(e^{.05t}S(t)) = .05e^{.05t}$

$e^{.05t}S(t) = e^{.05t} + C$

$S(t) = 1 + Ce^{-.05t}$

$2 = S(0) = 1 + C \Rightarrow C = 1$

$\Rightarrow S(t) = 1 + e^{-.05t}$

Never

$S(t) > 1$

always.

$1\frac{1}{2}$

$12\frac{1}{2}$

5. First find the solution to $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 80y = 0$, y(0) = 0, y'(0) = 1.

$2r^2 + 2r - 80 = 0$

$2(r+10)(r-8) = 0$

$r = -10,\ r = 8$

$y(x) = c_1e^{-10x} + c_2e^{8x}$

$y'(x) = -10c_1e^{-10x} + 8c_2e^{8x}$

$0 = c_1 + c_2 \Rightarrow c_1 = -c_2$

$1 = 10c_2 + 8c_2$

$c_2 = \frac{1}{18}$

$c_1 = -\frac{1}{18}$

$y(x) = -\frac{1}{18}e^{-10x} + \frac{1}{18}e^{8x}$

$12\frac{1}{2}$

2

6. Find the value of k so that $f(x) = x^{-3} + kx^{-4}$ is a probability density function on $[1,+\infty)$ and then find the value of the mean for the probability density function.

$$1 = \int_1^{+\infty} (x^{-3} + kx^{-4})\,dx = \lim_{b\to+\infty} \left(\frac{x^{-2}}{-2} + \frac{kx^{-3}}{-3}\right)\Big|_1^b = \frac{1}{2} + k\frac{1}{3} \quad 2$$

2 2

$$\frac{1}{2} = k\,\frac{1}{3}$$

$$k = \frac{3}{2}$$

$$\text{mean} = \int_1^{+\infty} (x^{-2} + \tfrac{3}{2}x^{-3})\,dx = \lim_{b\to+\infty}\left(-\frac{1}{x} - \frac{3}{4}x^{-2}\right)\Big|_1^b$$

1

$$= 1 + \frac{3}{4} = \boxed{1.75} \quad \frac{1}{2}$$

7. Find the area of the surface generated by rotating about the x-axis the graph of y = sin(2x) from 0 to π/4.

$$\int_0^{\frac{\pi}{4}} 2\pi \sin(2x)\sqrt{1+(\cos(2x)\,2)^2}\,dx$$

1 2 2 $\frac{1}{2}$

$$\int_0^{\frac{\pi}{4}} 2\pi \sin(2x)\sqrt{1+4\cos^2(2x)}\,dx$$

$w = \cos(2x)$

$\frac{dw}{dx} = -2\sin(2x)$

$-dw = 2\sin(2x)$

$u = 2w$

$\frac{du}{dw} = 2 \quad \frac{1}{2}du = dw$

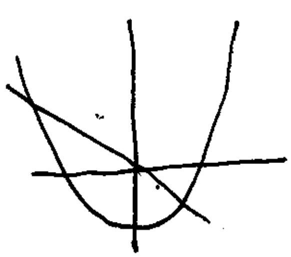
$$= \int_1^0 -\sqrt{1+4w^2}\,dw = \int_0^1 \sqrt{1+4w^2}\,dw$$

$$= \int_0^2 \sqrt{1+u^2}\,\tfrac{1}{2}du = \frac{1}{2}\int_0^2 \sqrt{1+u^2}\,du$$

$$= \frac{1}{2}\left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln(u+\sqrt{1+u^2})\right]_0^2$$

$$= \frac{1}{2}\left[\sqrt{5} + \frac{1}{2}\ln(2+\sqrt{5})\right]$$

8. Let A be the region bounded by $y = -2x$ and $y = x^2-3$. Suppose A has a uniform mass density ρ. Find the moment about the x-axis and the moment about the y-axis. You need only set up the integrals; you do not have to evaluate them. 2



2

$x^2 - 3 = -2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3,\ x = 1$

2

$$M_y = \int_{-3}^{1} x\left[-2x - x^2 + 3\right]dx$$

$$M_x = \int_{-3}^{1} \frac{1}{2}\left[(-2x)^2 - (x^2-3)^2\right]dx$$

$\frac{1}{2}$ 2