Test 1
MAT 162

Spring, 2014
Gurganus

Name: Key

Directions: Show all work for partial credit purposes. You may use a graphing calculator and notes recorded on one side of a single 8.5 by 11 inch paper. Otherwise the test is closed book. When you turn in your test, staple your notes to this sheet.

For 1-4, calculate the following:

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1. $\int xe^{7x}dx$ $= \frac{1}{7}xe^{7x} - \int \frac{1}{7}e^{7x}dx$ 2

$u = x \quad dv = e^{7x}dx$ 2 $\quad = \frac{1}{7}xe^{7x} - \frac{1}{49}e^{7x} + C$

$du = dx \quad v = \frac{1}{7}e^{7x}$ 2

2 2

2 2

2. $\int \cos^5(x)\sin^4(x)dx$ $= \int (1-\sin^2 x)^2 \sin^4 x \cos x \, dx$

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$= \int (1-u^2)^2 u^4 \, du$ 2

$= \int u^4 - 2u^6 + u^8 \, du$ 2

$= \frac{1}{5}u^5 - \frac{2}{7}u^7 + \frac{1}{9}u^9 + C$ 1

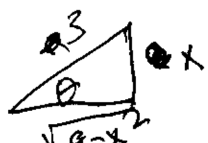
$= \frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + C$

10 3. $\int \frac{\sqrt{9-x^2}}{x^2}dx$ $= \int \frac{3\cos\theta}{3^2\sin^2\theta} 3\cos\theta \, d\theta$ 1

2 $x = 3\sin\theta$

$\sqrt{9-x^2} = \sqrt{9(1-\sin^2\theta)}$ $\quad = \int \cot^2\theta \, d\theta$ 1

1 $= 3\cos\theta$ $\quad = \int \csc^2\theta - 1 \, d\theta$ 1




$= -\cot\theta - \theta + C$ 1

1 $\quad = -\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right) + C$ 1

$dx = 3\cos\theta \, d\theta$

1

4. $\int \frac{1}{x^2+2x-35}dx$ $\frac{1}{(x+7)(x-5)} = \frac{A}{x+7} + \frac{B}{x-5} = -\frac{1}{12}\frac{1}{x+7} + \frac{1}{12}\frac{1}{x-5}$

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$1 = A(x-5) + B(x+7)$

$1 = B(12) \Rightarrow B = \frac{1}{12}$

$-\frac{1}{12}\ln|x+7| + \frac{1}{12}\ln|x-5| + C$ $A = -\frac{1}{12}$

2 2 2 2 2

5. Estimate $\int_4^7 \cos(x^2-1)dx$ using the Simpsons Rule with n = 8. Write the sum; you do not have to evaluate the sum.

8

$\Delta x = \frac{7-4}{8} = \frac{3}{8}$ 2 $\frac{3/8}{3}\left(f(4) + 4f() + 2f() + 4f() + 2f() + 4f() + 2f(4) + 4f() + f()\right)$

2 4 $4\frac{3}{8}$ $4\frac{3}{4}$ $5\frac{1}{8}$ 5.5 $5\frac{7}{8}$ $6\frac{1}{4}$ $6\frac{5}{8}$ 7

6. Calculate the following; it the integral does not converge, state "does not converge."

a. $\int_1^{+\infty} \frac{x}{x^2+1}dx$ $= \lim_{b\to+\infty} \int_1^b \frac{x}{x^2+1}dx = \lim_{b\to+\infty} \frac{1}{2}\ln(x^2+1)\Big|_1^b$ 2 2

$= \lim_{b\to+\infty} \frac{1}{2}\ln(b^2+1) - \frac{1}{2}\ln 2 = +\infty$ 2

8

"does not converge" 2

2

b. $\int_1^3 \frac{4}{(1-x^2)}dx$ $= \lim_{b\to1} \int_b^3 \left(\frac{2}{1-x} + \frac{2}{1+x}\right)dx$ 2

8

$\frac{4}{(1-x)(1+x)} = \frac{A}{1-x} + \frac{B}{1+x}$ $= \lim_{b\to1} -2\ln|1-x| + 2\ln|1+x|\Big|_b^3$

$4 = A(1+x) + B(1-x)$ 2 $= \lim_{b\to1} -2\ln|2| + 2\ln 4 + 2\ln|1-b| - 2\ln(1+b)$

$4 = A(2) \Rightarrow A = 2$ $B = +2$ $= \lim_{b\to1} 2\ln 2 + 2\ln\left|\frac{1-b}{1+b}\right| \Rightarrow +\infty$

diverges

7. Tell why the following converge or diverge:

a. $\int_1^{+\infty} \frac{x^2+1}{(x+1)^4}dx$

8 3 $\frac{x^2+1}{(x+1)^4} \le \frac{x^2+x^2}{(x)^4} = \frac{2x^2}{x^4} = \frac{2}{x^2}$

3 $2\int_1^{+\infty} \frac{1}{x^2}dx$ converges 2 converges

$\therefore \int_1^{+\infty} \frac{x^2+1}{(x+1)^4}dx$ converges

8 b. $\int_1^{+\infty} \frac{x^6+1}{2x^7-1}dx$

3 $\frac{1}{2}\frac{1}{x} = \frac{x^6}{2x^7-x^7}$ ~~$\le$~~ $\frac{x^6+1}{2x^7-1}$

3 $\frac{1}{2}\int_1^{+\infty} \frac{1}{x}dx$ diverges

$\therefore \int_1^{+\infty} \frac{x^6+1}{2x^7-1}dx$ diverges

2

8. Calculate $\int \frac{2x+5}{x^2+2x+3\cancel{5}7} dx$

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$2 \int \frac{2x+5}{(x+1)^2+6^2}$

$\int \frac{2(x+1)+3}{(x+1)^2+6^2} dx = \int \frac{2(x+1)}{(x+1)^2+6^2} dx + 3\int \frac{1}{(x+1)^2+6^2} dx$ 1

2 1 1

$= 1 \ln (x+1)^2+6^2 + \frac{3}{6} \arctan \frac{x+1}{6} + C$ 1

$= 1 \ln (x^2+2x+37) + \frac{1}{2} \arctan \frac{x+1}{6} + C$

1

9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the integral): $\int \frac{4x+5}{(x^2-12x-28)^2(x^2+2x+21)^3} dx$

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$\frac{4x+5}{(x-14)^2(x+2)^2\left[(x+1)^2+(\sqrt{20})^2\right]^3}$ 2

1

$\frac{A_1}{(x-14)} + \frac{A_2}{(x-14)^2} + \frac{B_1}{(x+2)} + \frac{B_2}{(x+2)^2} + \frac{C_1x+D_1}{x^2+2x+21} + \frac{C_2x+D_2}{(x^2+2x+21)^2} + \frac{C_3x+D_3}{(x^2+2x+21)^3}$

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