(27) $\int e^x \cos x\,dx = e^x \sin x - \int (\sin x)\, e^x\,dx$

$= e^x \sin x - \left[\int e^x \sin x\,dx\right] = e^x \sin x - \left[e^x(-\cos x) - \int -\cos x\, e^x\,dx\right]$

$= e^x \sin x + e^x \cos x - \int \cos x\, e^x\,dx$

$2\int \cos x\, e^x\,dx = e^x \sin x + e^x \cos x$

$\boxed{\int e^x \cos x\,dx = \frac{1}{2}\left(e^x \sin x + e^x \cos x\right) + C}$

(28) $\dfrac{3x^3 - x^2 + 6x - 4}{(x^2+1)(x^2+2)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{x^2+2}$

$3x^3 - x^2 + 6x - 4 = (Ax+B)(x^2+2) + (Cx+D)(x^2+1)$

$= (A+C)x^3 + (B+D)x^2 + (2A+C)x + (2B+D)$

$$\begin{array}{ll} e_1 & A + C = 3 \\ e_2 & B + D = -1 \\ e_3 & 2A + C = 6 \\ e_4 & 2B + D = -4 \end{array}$$

$e_3 - e_1$: $A = 3$

e_1: $3 + C = 3 \Rightarrow C = 0$

$e_4 - e_2$: $B = -3$

e_2: $-3 + D = -1 \Rightarrow D = 2$

$\int \dfrac{3x^3 - x^2 + 6x - 4}{(x^2+1)(x^2+2)}\,dx = \int \dfrac{3x-3}{x^2+1}\,dx + \int \dfrac{2}{x^2+2}\,dx$

$= \dfrac{3}{2}\int \dfrac{2x}{x^2+1}\,dx - 3\int \dfrac{1}{x^2+1}\,dx + 2\int \dfrac{1}{x^2+2}\,dx$

$= \dfrac{3}{2}\ln(x^2+1) - 3\arctan x + \dfrac{2}{\sqrt{2}}\arctan\left(\dfrac{x}{\sqrt{2}}\right) + C$

$= \boxed{\dfrac{3}{2}\ln(x^2+1) - 3\arctan x + \sqrt{2}\arctan\left(\dfrac{x}{\sqrt{2}}\right) + C}$

(26) $\int x \sin x \cos x\,dx = \int x \dfrac{\sin 2x}{2}\,dx$

$= x\left(\dfrac{-\cos 2x}{4}\right) - \int \left(\dfrac{-\cos 2x}{4}\right)dx$

$= -\frac{1}{4}x\cos 2x + \frac{1}{4}\int \cos 2x\,dx$

$= -\frac{1}{4}x\cos 2x + \frac{1}{4}\dfrac{\sin 2x}{2} + C$

$= \boxed{-\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C}$

(29) $\int_0^{\frac{\pi}{2}} \cos^3 x \sin 2x \, dx = \int_0^{\frac{\pi}{2}} \cos^3 x \, 2 \sin x \cos x \, dx$

$= 2\int_0^{\frac{\pi}{2}} \cos^4 x \sin x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-du = \sin x \, dx$

$= 2\int_1^0 u^4 (-du)$

$= \frac{-2}{5} u^5 \Big|_1^0 = \boxed{\frac{2}{5}}$

(28) $\int \frac{\sqrt[3]{x}+1}{\sqrt[3]{x}-1}\, dx = \int \frac{u+1}{u-1} 3u^2 \, du$

$u = x^{\frac{1}{3}}$

$u^3 = x$

$dx = 3u^2 \, du$

$= \int \frac{u-1+2}{u-1} 3u^2 \, du$

$= \int \left[1 + \frac{2}{u-1}\right] 3((u-1)+1)^2 \, du$

$= \int \left(1 + \frac{2}{u-1}\right)(3)\left[(u-1)^2 + 2(u-1) + 1\right] du$

$= (3)\int (u-1)^2 + 2(u-1) + 1 + 2(u-1) + 2 + \frac{1}{u-1} \, du$

$= 3\int \left[(u-1)^2 + 4(u-1) + 3 + \frac{1}{u-1}\right] du$

$= 3\left[\frac{(u-1)^3}{3} + 2(u-1)^2 + 3u + \ln|u-1|\right] + C$

$= (u-1)^3 + 6(u-1)^2 + 9u + 3\ln|u-1| + C$

$= \boxed{(x^{\frac{1}{3}}-1)^3 + 6(x^{\frac{1}{3}}-1)^2 + 9x^{\frac{1}{3}} + 3\ln|x^{\frac{1}{3}}-1| + C}$

Alternative!

$$\begin{array}{r} 3u^2 + 6u + 6 \\ u-1 \overline{) \, 3u^3 + 3u^2} \\ \underline{3u^3 - 3u^2} \\ 6u^2 \\ \underline{6u^2 - 6u} \\ 6u \\ \underline{6u - 6} \\ 6 \end{array}$$

$\int \left[3u^2 + 6u + 6 + \frac{6}{u-1}\right] du$

$= u^3 + 3u^2 + 6u + 6\ln|u-1| + C$

$= \boxed{x + 3x^{3/2} + 6x^{\frac{1}{3}} + 6\ln|x^{\frac{1}{3}}-1| + C}$

(31) $u = \sqrt{e^x - 1}$; $u^2 = e^x - 1$; $e^x = u^2 + 1$; $e^x + 8 = u^2 + 9$

$$\frac{du}{dx} = \frac{1}{2}(e^x - 1)^{-\frac{1}{2}} e^x$$

$$= \frac{1}{2}\frac{1}{u}\, u^2 + 1 \Rightarrow dx = \frac{2u}{u^2+1}\, du$$

$$\int_0^{\ln 10} \frac{e^x \sqrt{e^x - 1}}{e^x + 8}\, dx = \int_0^3 \frac{(u^2+1)u}{u^2+9}\, \frac{2u}{u^2+1}\, du$$

$$= \int_0^3 \frac{2u^2}{u^2+9}\, du$$

$$= \int_0^3 \frac{2(u^2+9-9)}{u^2+9}\, du$$

$$= \int_0^3 2\left[1 - \frac{9}{u^2+9}\right] du$$

$$= 2\left[u - 9\,\frac{1}{3}\arctan\frac{u}{3}\right]\Bigg|_0^3$$

$$= 2\left[3 - 3\arctan 1\right] = 2\left[3 - 3\,\frac{\pi}{4}\right] = \boxed{6 - \frac{3}{2}\pi}$$

(32) $$\int_0^{\frac{\pi}{4}} \frac{x \sin x}{\cos^3 x}\, dx = +\frac{1}{2} x \sec^2 x \Big|_0^{\pi/4} - \int_0^{\pi/4} \frac{1}{2}\sec^2 x\, dx$$

$u = x$ $\quad dv = \frac{\sin x}{\cos^3 x}\, dx$

$du = dx$ $\quad \frac{dv}{dx} = \frac{\sin x}{\cos^3 x}$

$v = +\frac{1}{2}(\cos x)^{-2}$

$= +\frac{1}{2}\sec^2 x$

$$= +\frac{1}{2} x \sec^2 x \Big|_0^{\frac{\pi}{4}} - \frac{1}{2}(\tan x)\Big|_0^{\pi/4}$$

$$= +\frac{1}{2}\,\frac{\pi}{4}\,\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} - \frac{1}{2}\left(\tan \pi/4 - \tan 0\right)$$

$$= +\frac{1}{2}\,\frac{\pi}{4}\,\frac{1}{\frac{1}{2}} - \frac{1}{2}(1)$$

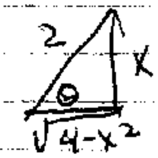
$$= +\frac{\pi}{4} - \frac{1}{2} = \boxed{\frac{\pi}{4} - \frac{1}{2}}$$

(33) $x = 2\sin\theta \Rightarrow (4-x^2)^{3/2} = (2\cos\theta)^3 = 8\cos^3\theta$

$dx = 2\cos\theta\, d\theta$

$$\int \frac{x^2}{(4-x^2)^{3/2}}dx = \int \frac{4\sin^2\theta}{8\cos^3\theta}\, 2\cos\theta\, d\theta = \int \tan^2\theta\, d\theta$$

$$= \int(\sec^2\theta - 1)\,d\theta = \tan\theta - \theta + C$$



$$= \boxed{\frac{x}{\sqrt{4-x^2}} - \sin^{-1}\left(\frac{x}{2}\right) + C}$$

(34) $u = (\arcsin x)^2 \quad dv = dx$

$$\int(\arcsin x)^2\,dx = x(\arcsin x)^2 - \int 2x\arcsin x\,\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= x(\arcsin x)^2 - 2\underline{\int \arcsin x\,\frac{x}{\sqrt{1-x^2}}\,dx}$$

$$\int \arcsin x\,\frac{x}{\sqrt{1-x^2}}\,dx = -(\arcsin x)(1-x^2)^{\frac12} - \int -(1-x^2)^{\frac12}\,\frac{1}{\sqrt{1-x^2}}\,dx$$

$w = \arcsin x \qquad dz = \frac{x}{\sqrt{1-x^2}}\,dx$

$dw = \frac{1}{\sqrt{1-x^2}}\,dx$

$z = -(1-x^2)^{\frac12}$

$$= -(\arcsin x)(1-x^2)^{\frac12} + \int dx$$

$$= -(\arcsin x)(1-x^2)^{\frac12} + x + C$$

$$\therefore \boxed{\int(\arcsin x)^2\,dx = x(\arcsin x)^2 + 2\arcsin x\,(1-x^2)^{\frac12} - 2x + C}$$

(35) $u = xe^{2x} \Rightarrow du = (e^{2x} + 2xe^{2x})\,dx = e^{2x}(1+2x)\,dx$

$dv = \frac{1}{(1+2x)^2}\,dx \Rightarrow v = -\frac12\,\frac{1}{1+2x} \Rightarrow v\,du = -\frac12 e^{2x}\,dx$

$$\int_0^{\frac12} \frac{xe^{2x}}{(1+2x)^2} = -\frac12 xe^{2x}\frac{1}{1+2x}\Big|_0^{\frac12} - \int_0^{\frac12} -\frac12 e^{2x}\,dx$$

$$= \left[-\frac12 xe^{2x}\frac{1}{1+2x} + \frac14 e^{2x}\right]_0^{\frac12} = e^{2x}\left[-\frac12\,\frac{x}{1+2x} + \frac14\right]\Big|_0^{\frac12}$$

$$= e\left[-\frac12\,\frac{\frac12}{1+1} + \frac14\right] - 1\left(\frac14\right) = \boxed{\frac{e}{8} - \frac14}$$