Test 3
MAT 162

Sum II, 2012 Name: Key
Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book.

1. For the parametric curve, $x = t^2 + 4t$, $y = t^3 + 4.5t^2$, $-5 \le t \le 2$:

(6) (a) Calculate the following: $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$.

$\frac{dx}{dt} = 2t+4$ 1 ; $\frac{dy}{dt} = 3t^2 + 9t$ 1 ; $\frac{dy}{dx} = \frac{3t^2+9t}{2t+4}$ 2 ; $\frac{d^2y}{dx^2} = \dfrac{\frac{(2t+4)(6t+9)-(3t^2+9t)2}{(2t+4)^2}}{2t+4}$ 2

6 (b) Find when x is increasing and decreasing.

$\frac{dx}{dt} = 2(t+2)$ $t+2$: $-$ | -2 | $+$

2 reasons

x is dec when $t \in [-5,-2]$ 2

x is inc when $t \in [-2,2]$ 2

6 (c) Tell when y is increasing and decreasing.

$\frac{dy}{dt} = 3t(t+3)$

2 reasons

$t+3$: $-$ | -3 | $+$ | 0 | $+$
t: $-$ | | $-$ | | $+$
$+$ | -3 | $-$ | 0 | $+$

(2) y is increasing for $t \in [-5,-3] \cup [0, 2]$

(2) y is decreasing for $t \in [-3,0]$

6 (d) Find the xy coordinates where there is a horizontal tangent line.

Answers are reversed in d & e

$\frac{dx}{dt} = 2t+4 = 0$
$t = -2$ 2

For $t=-2$ $x = (-2)^2 + 4(-2) = -4$ 2
$y = (-2)^3 + 4.5(-2)^2 = -8 + 18 = 10$

2 $\boxed{(-4, 10)}$ Vertical tan line

6 (e) Are there any vertical tangent lines? If so, what are the xy coordinates where there is a vertical tangent line?

$\frac{dy}{dt} = 3t(t+3) = 0 \Rightarrow t=-3, t=0$ 2

For $t=-3$, $x = (-3)^2 + 4(-3) = 9-12 = -3$
$y = -27 + 4.5(9) = 13.5$ 2

$(-3, 13.5)$

For $t=0$, $x=y=0$ 2

$(0,0)$

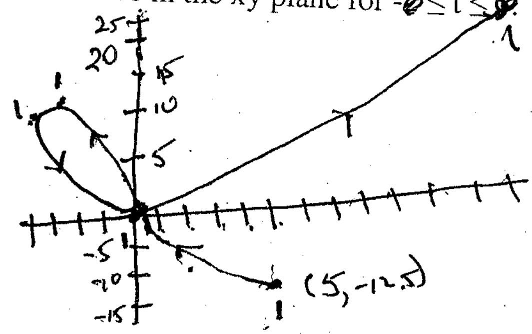
30

(f) Sketch the graph of the parametric curve in the xy plane for $-5 \le t \le 2$ (5, 2 corrected in class)

6

t	x	y	
-5	5	-12.5	start
-3	-3	13.5	hori
-2	-4	[illegible]	vertical
0	0	0	hor
2	12	28	final

graph should reflect these points



(g) Write down, but do not evaluate the integral of the length of the curve.

4 $L = \int_{-5}^{2} \sqrt{(2t+4)^2 + (3t^2+9t)^2}\, dt$

(h) (Extra credit—do not attempt until finished with all other problems.) Find when y = f(x) is concave up and concave down.

10 points

$$\frac{d^2y}{dx^2} = \frac{6t^2+24t+36}{(2t+4)^3} = \frac{6(t^2+4t+6)}{2^3(t+2)^3} = \frac{6(t^2+4t+4+2)}{2^3(t+2)^3}$$

$$= \boxed{\frac{3}{4}\,\frac{[(t+2)^2+2]}{(t+2)^3}}$$ 5

The sign of $\frac{d^2y}{dx^2}$ is the same as the sign of $t+2$

$t+2$ — +, at -2

2 $y = f(x)$ is concave down when $t < -2$

2 $y = f(x)$ is concave up when $t > -2$.

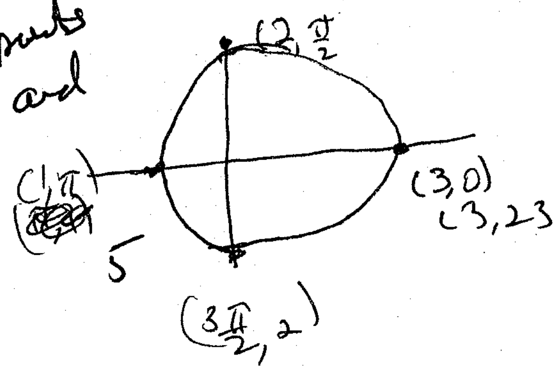
10 w/o extra credit, 20 with extra credit

2. (a) Sketch the polar graph of $r = 2 + \cos(\theta)$ from $\theta = 0$ to 2π.. Identify in polar coordinates where the graph crosses the x-axis and y-axis.

θ	r	
0	$2+1$	$= 3$
$\frac{\pi}{2}$	$2+0$	2
π	$2-1$	1
$\frac{3\pi}{2}$	$2+0$	2
2π	$2+1$	3

These points and

10



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(b) Find the area between the curve and the origin in the first quadrant.

$$A = \int_0^{\frac{\pi}{2}} \frac{1}{2}(2+\cos\theta)^2 \, d\theta \quad 2$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}(4 + 4\cos\theta + \cos^2\theta) \, d\theta \quad 1$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(4 + 4\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta \quad 1$$

6

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (4.5 + 4\cos\theta + \frac{1}{2}\cos 2\theta) \, d\theta$$

$$= \frac{1}{2}\left(4.5\theta \;\; 4\sin\theta + \frac{1}{4}\sin 2\theta\right)\Big|_0^{\frac{\pi}{2}} \quad 1$$

$$= \frac{1}{2}\left(4.5\frac{\pi}{2} + 4\right) = \frac{4.5}{4}\pi + 2 = 5.534291725$$

(c) Write down, but do not evaluate the integral of the length of this curve.

$$L = \int_0^{2\pi} \sqrt{(2+\cos\theta)^2 + (\sin\theta)^2} \, d\theta$$

4

20 points

III. For $y^2 - 32x - 10y + 27 = 0$,

(a) Identify the conic section.

$y^2 - 10y + 27 = 32x$

(4) $y^2 - 10y + 25 + 2 = 32x$

$(y-5)^2 = 32x - 2$

$\frac{1}{32}(y-5)^2 = x - \frac{1}{16}$ or $x - \frac{1}{16} = \frac{1}{4(8)}(y-5)^2$

parabola

(b) Find the center.

3 $(\frac{1}{16}, 5)$

(c) Find the vertices.

3 $(\frac{1}{16}, 5)$

(d) Find the foci.

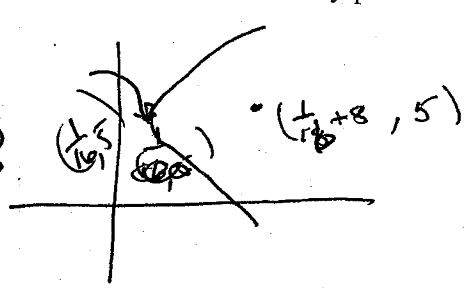
3 $(\frac{1}{16} + 8, 5)$

(e) Find the eccentricity.

3 $e = 1$ because parabola

(f) Sketch the curve in the xy plane.

3



20 points

IV. (a) Rewrite the polar equation $r = \dfrac{15}{1+4\sin(\theta)}$ using xy coordinates.

4 (2) $r + 4r\sin\theta = 15$

$r = 15 - 4r\sin\theta$

$r^2 = (15 - 4r\sin\theta)^2$

anything (2) from here down is OK for (a)

$x^2+y^2 = (15-4y)^2$

$x^2+y^2 = 15^2 - 120y + 16y^2$

$x^2 = 15^2 - 120y + 15y^2 = 15^2 + 15(y^2 - 8y)$

$x^2 = 15^2 + 15(y^2 - 8y + 16) - 15(16) = 15(y-4)^2 - 15$

$x^2 - 15(y-4)^2 = -15$

$(y-4)^2 - \frac{1}{15}x^2 = 1$

Hyperbola

(b) Calculate the eccentricity, center, foci, and vertices of the curve.

$a^2 = 1$

$b^2 = 15$

$c^2 = a^2 + b^2 = 1 + 15 = 16$

4

$c = 4$

$e = \frac{c}{a} = \boxed{4}$ or 4

center $= (0,4)$

foci $= (0, 4 \pm c) = (0, 4 \pm 4) = \begin{cases} (0,8) \\ (0,0) \end{cases}$

vertices $= (0, 4 \pm a) = (0, 4 \pm 1) = \begin{cases} (0,5) \\ (0,3) \end{cases}$

(c) Sketch the graph of $r = \dfrac{15}{1+4\sin(\theta)}$.

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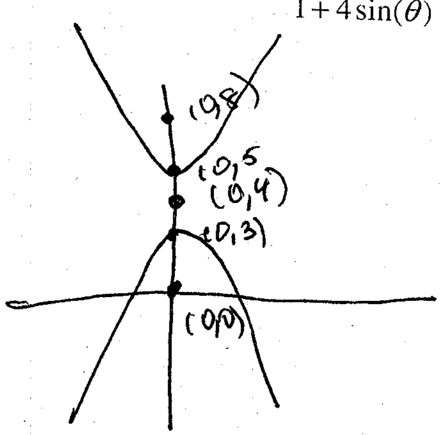

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