Test 3 | Sum II, 2011 Name: Key

MAT 162 | Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book.

1. For the parametric curve, $x = 2t^3$, $y = t^2 - t$, $-2 \le t \le 2$:

(a) Calculate the following: $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$.

$\frac{dx}{dt} = 6t^2$; $\frac{dy}{dt} = 2t-1$; $\frac{dy}{dx} = \frac{2t-1}{6t^2}$; $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{6t^2(2)-(2t-1)(12t)}{(6t^2)^2}}{6t^2}$

$= \frac{12t-12t^2}{(6t^2)^3} = \frac{6\cdot 2(t-t^2)}{6^3 t^6}$

$= \frac{1}{18}\frac{1-t}{t^5}$

(b) Tell when x is increasing. Tell when y is decreasing. Tell when y = f(x) is concave up.

X is inc. $\frac{dx}{dt} = 6t^2 > 0$ for $-2 \le t \le 2$

y is dec. $\frac{dy}{dt} = 2t-1 < 0$ for $-2 \le t \le \frac{1}{2}$

$0 < \frac{d^2y}{dx^2} = -\frac{1}{18}\frac{t-1}{t^5}$

$-\frac{1}{18}$	−	−	−
t^5	−	+	+
$t-1$	−	−	+
	−	(+)	−
	0		1

f(x) is concave up on (0, 1]

(c) Tell where y = f(x) has a horizontal tangent line. Are there any vertical tangent lines.? If so, where?

y = f(x) has a horizontal tangent line when $\frac{dy}{dt} = 0$ or $t = \frac{1}{2}$; $x = \frac{1}{4}$, $y = -\frac{1}{4}$

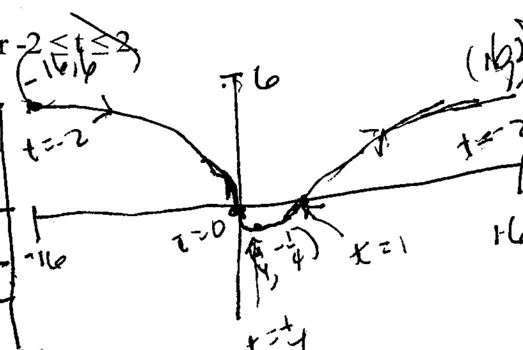
y = f(x) has a vertical tangent line when $0 = \frac{dx}{dt} = 6t^2$ or $t = 0$ $x = 0, y = 0$

(d) Sketch the graph of the parametric curve in the xy plane for $-2 \le t \le 2$.

x	inc	inc	inc	inc
y	dec	dec	inc	inc
concavity	down	up	up	down

t: −2, 0, ½, 1, 2

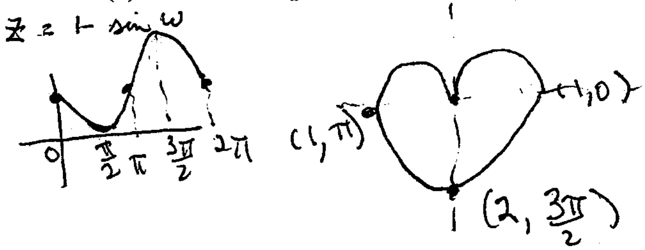
t	x	y
-2	-16	6
0	0	0
$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$
1	2	0
2	16	2



(e) Write down, but do not evaluate the integral of the length of the curve.

$L = \int_{-2}^{2} \sqrt{(6t^2)^2 + (2t-1)^2}\, dt$

2. (a) Sketch the polar graph of r = 1 – sin(θ) from θ = 0 to 2π..



(b) Find the area between the curve and the origin.

$$A = \int_0^{2\pi} \frac{1}{2}(1-\sin(\theta))^2 d\theta = \frac{1}{2}\int_0^{2\pi}(1-2\sin\theta+\sin^2\theta)$$

$$= \frac{1}{2}\int_0^{2\pi}\left(\frac{3}{2}-2\sin\theta - \frac{\cos 2\theta}{2}\right)d\theta$$

$$= \frac{1}{2}\left[\frac{3}{2}\theta + 2\cos\theta - \frac{\sin 2\theta}{4}\right]\Big|_0^{2\pi} = \frac{1}{2}\left[\frac{3}{2}2\pi + 2 - (0+2)\right]$$

$$= \frac{3}{2}\pi \approx 4.71239$$

(c) Write down, but do not evaluate the integral of the length of this curve.

$$L = \int_0^{2\pi}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta = \int_0^{2\pi}\sqrt{(1-\sin\theta)^2+(-\cos\theta)^2}\,d\theta$$

3. For $25x^2 + 16y^2 - 150x + 64y - 111 = 0$,

(a) Identify the conic section.

$25(x^2-6x) + 16(y^2+4y) = 111$

$25(x^2-6x+9) + 16(y^2+4y+4) = 111+64+225 = 400$

$$\frac{(x-3)^2}{16} + \frac{(y+2)^2}{25} = 1$$

(b) Find the center.

$(3,-2)$

(c) Find the vertices.

$(3, -2\pm 5)$ $(3\pm 4, -2)$

$(3, 3)$ $(3,-7)$ $(7,-2)$ $(-1,-2)$

(d) Find the foci.

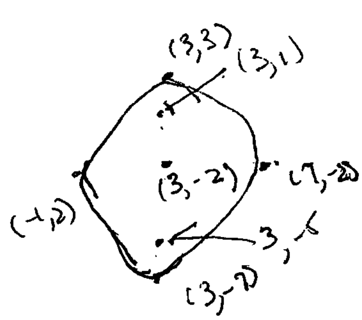
$c^2 = a^2 - b^2 = 25-16 = 9$ $(3, -2\pm 3)$ → $(3,1)$, $(3,-5)$ foci

$c = \pm 3$

(e) Find the eccentricity.

$e = \frac{c}{a} = \frac{3}{5} = .6$

(f) Sketch the curve in the xy plane.



4. (a) Rewrite the polar equation $r = \dfrac{3}{2+2\sin(\theta)}$ using xy coordinates.

$2r + 2r\sin\theta = 3$

$2r = 3 - 2r\sin\theta$

$4r^2 = (3 - 2r\sin\theta)^2$

$4(x^2+y^2) = (3-2y)^2 = 9 - 12y + 4y^2$

$4x^2 = 9 - 12y = -12(y - \frac{9}{12})$

$x^2 = -3(y - \frac{9}{12})$

$y - \frac{9}{12} = y - \frac{3}{4} = -\frac{1}{3}x^2 = \frac{1}{4(-\frac{3}{4})}x^2$

any of these are acceptable for 4(a)

$\frac{1}{4p} = -\frac{1}{3}$

$4p = -3$

$p = \frac{-3}{4}$

(b) Calculate the eccentricity, center, foci, and vertices of the curve.

$e = 1$

center $= (h,k) = (0, \frac{3}{4})$

vertex $= (0, \frac{3}{4})$

focus $= (0,0)$

(c) Sketch the graph of $r = \dfrac{3}{2+2\sin(\theta)}$.