3. Find y(x), the solution to $\frac{dy}{dx} = x^{-1}y+x$, $y(1) = 3$.

$$\frac{dy}{dx} - \frac{1}{x}y = x$$

$$\mu(x) = e^{\int -\frac{1}{x}} = e^{-\ln x} = \frac{1}{x}$$

$$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = x\left(\frac{1}{x}\right) = 1$$

$$\frac{d}{dx}\left(\frac{1}{x}y\right) = 1$$

$$\frac{1}{x}y = x + C$$

$$y = x(x+C)$$

$$3 = y(1) = 1 + C \Rightarrow C = 2$$

$$\boxed{y = x(x+2) = x^2 + 2x}$$

4. Find y(x), the solution to $\frac{dy}{dx} = x^2(\cos y)^2$, $y(0) = \pi/4$.

$$\frac{dy}{dx} = x^2(\cos y)^2$$

$$\sec^2 y\, dy = x^2\, dx$$

$$\int \sec^2 y\, dy = \int x^2\, dx$$

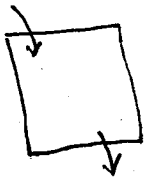
$$\tan y = \frac{1}{3}x^3 + C$$

$$1 = \tan\frac{\pi}{4} = \frac{1}{3}(0^3) + C$$

$$\tan y = \frac{1}{3}x^3 + 1$$

$$\boxed{y = \arctan\left(\frac{1}{3}x^3 + 1\right)}$$

5. A tank is filled with 300 liters of contaminated water containing 3 kg of toxins. Pure water is pumped in at a rate of 40 l/min., mixes instantaneously, and then is pumped out at the same rate. Find y(t) the number of grams of the toxin in the tank t minutes after the rinse begins. Then find the time at which there is .01 kg of toxin present.



$y(t)$ = amount of toxin in tank at time t

$y(0) = 3$

$\frac{dy}{dt}$ = rate in − rate out

$= 0 - \frac{y}{300}(40)$ kg/min.

$\frac{dy}{dt} = -\frac{2}{15}y$, $y(0) = 3$

$y(t) = 3e^{-\frac{2}{15}t}$

Find t so that $y(t) = 3e^{-\frac{2}{15}t} = .01$

$e^{-\frac{2}{15}t} = \frac{.01}{3}$

$-\frac{2}{15}t = \ln\frac{.01}{3}$

$t = -\frac{15}{2}\ln\frac{.01}{3}$

$= 42.8$ minutes

6. First find the solution to $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 21y = 0$, $y(0) = 1$, $y'(0) = 2$.

$r^2 - 4r - 21 = 0$

$(r-7)(r+3) = 0$

$r = 7 \quad r = -3$

$y(x) = c_1e^{7x} + c_2e^{-3x}$; $y'(x) = 7c_1e^{7x} - 3c_2e^{-3x}$

$1 = y(0) = c_1 + c_2$

$2 = y'(0) = 7c_1 - 3c_2$ $\Rightarrow$ $3 = 3c_1 + 3c_2$, $2 = 7c_1 - 3c_2$

$5 = 10c_1$

$c_1 = \frac{1}{2} \Rightarrow c_2 = \frac{1}{2}$

$\therefore y(x) = \frac{1}{2}e^{7x} + \frac{1}{2}e^{-3x}$