Test 2
MAT 162

S II, 2011
Gurganus

Name: Key

Directions: Show all work for partial credit purposes. You may use a graphing calculator. The test is closed book.

1. Use Euler's Method to approximate y(2.2) given $\frac{dy}{dx} = 5x - 3y$, and y(2) = 1. Use a stepsize of 0.1

(10) points

x	y	$F(x,y) \cdot .1$
2	1	.7
2.1	1.7	.54
2.2	2.24	

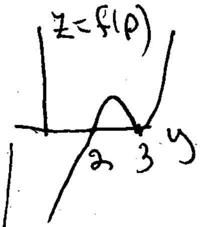
where $F(x,y) = 5x - 3y$

$F(2,1) = 10 - 3 = 7$

$F(2.1, 1.7) = 5(2.1) - 3(1.7)$

$= 10.5 - 5.1 = 5.4$

2. A. Find the equilibrium solutions to $\frac{dy}{dx} = (y-2)(y-3)^2$.

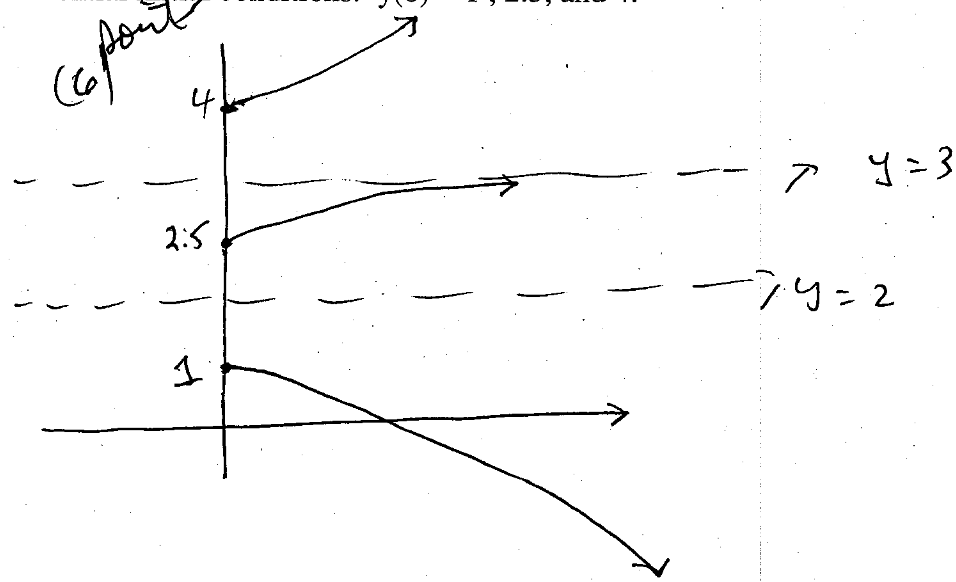


$\boxed{\begin{matrix} y = 2 \\ y = 3 \end{matrix}}$

4 points

B. On a single graph, sketch three solutions to $\frac{dy}{dx} = (y-2)(y-3)^2$ for the three differential initial conditions: y(0) = 1 , 2.5, and 4.

(6 points



7. Find the value of k so that $f(x) = x^{-10} + kx^{-11}$ is a probability density function on $[1,+\infty)$ and then find the value of the mean for the probability density function.

$$1 = \int_1^{+\infty} (x^{-10} + k x^{-11})\,dx = \lim_{b \to +\infty} \int_1^b (x^{-10} + k x^{-11})\,dx$$

$$= \lim_{b \to +\infty} \left(\frac{x^{-9}}{-9} + \frac{k x^{-10}}{-10} \right)\Big|_1^b = \lim_{b \to +\infty} \frac{1}{-9b^9} - \frac{k}{10b^{10}} + \frac{1}{9} + \frac{k}{10})$$

10

$$= \frac{1}{9} + \frac{k}{10} \Rightarrow \frac{8}{9} = \frac{k}{10} \Rightarrow \boxed{k = \frac{80}{9}}$$

$$\mu = \int_1^{+\infty} x(x^{-10} + kx^{-11})\,dx = \int_1^{+\infty} (x^{-9} + kx^{-10})\,dx =$$

10

$$\lim_{b \to +\infty} \frac{x^{-8}}{-8} + \frac{kx^{-9}}{-9}\Big|_1^b = \frac{1}{8} + k\frac{1}{9} = \frac{1}{8} + \frac{1}{9}\frac{80}{9} = \boxed{\frac{1}{8} + \frac{80}{81}}$$

8. For $f(x) = 17 + 6x^{1.5}$, find the length of the curve $y = f(x)$ from $x = 2$ to 8.

$$s = \int_2^8 \sqrt{1 + (f'(x))^2}\,dx$$

$$= \int_2^8 \sqrt{1 + (9x^{.5})^2}\,dx = \int_2^8 \sqrt{1 + 81x}\,dx$$

10

$$\left(\frac{2}{3}(1+81x)^{3/2}\frac{1}{81}\right)\Big|_2^8$$

$$= \frac{2}{243}\left[(1 + 81(8))^{3/2} - (1 + 81(2))^{3/2}\right]$$

9. Find the area of the surface generated by rotating about the x-axis the graph of y = cos(x) from 0 to π/4.

(10)

$$S = \int_0^{\frac{\pi}{4}} 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx = \int_0^{\frac{\pi}{4}} 2\pi \cos x\sqrt{1+\sin^2 x}\,dx$$

$u = \sin x$

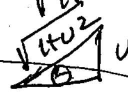
$du = \cos x\,dx$

$$= \int_0^{\frac{\sqrt{2}}{2}} 2\pi\sqrt{1+u^2}\,du$$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$\sqrt{1+u^2} = \sec\theta$



$$= \int 2\pi \sec^3\theta\,d\theta = \int 2\pi \sec\theta\sec^2\theta\,d\theta \qquad w = \sec\theta \quad dv = \sec^2\theta\,d\theta$$

$dw = \sec\theta\tan\theta\,d\theta \quad v = \tan\theta$

9 points if this far

$$= 2\pi\left[\sec\theta\tan\theta - \int\sec\theta\tan^2\theta\,d\theta\right]$$

$$= 2\pi\left[\sec\theta\tan\theta - \int\sec\theta(\sec^2\theta-1)\,d\theta\right]$$

$$= 2\pi\left[\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|\right] - \int\sec^3\theta\,d\theta$$

$$= 2\pi\left[\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|\right|_0^{\frac{\sqrt{2}}{2}}$$

$$= \pi\left(\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|\right)\Big|_0$$

$$= \pi\left(u\sqrt{1+u^2} + \ln\left|u+\sqrt{1+u^2}\right|\right)$$

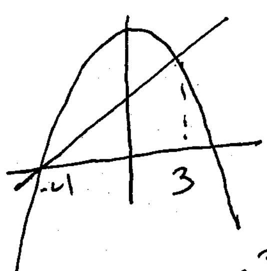
$$= \pi\left(\frac{\sqrt{2}}{2}\sqrt{1+\tfrac{1}{2}} + \ln\left(\sqrt{1+\tfrac{1}{2}} + \frac{\sqrt{2}}{2}\right)\right)$$

$$= \pi\left(\frac{\sqrt{3}}{2} + \ln\left(\sqrt{1.5} + \frac{\sqrt{3}}{2}\right)\right) = \boxed{4.7894}$$

The integral in #9 is not one that I expect to be done completely w/o notes on the test

10. Let A be the region bounded by y = x+4 and y = 16-x². Suppose A has a uniform mass density ρ. Find the moment about the x-axis and the moment about the y-axis.

(10)



$y = 16-x^2$

$y = x+4$

$16-x^2 = x+4$

$0 = x^2+x-12$

$0 = (x+4)(x-3)$

$$M_y = \int_{-4}^{3} \rho x(16-x^2-(x+4))\,dx = \rho\int_{-4}^{3}(12x-x^3-x^2)\,dx$$

$$= \rho\left(6x^2-\tfrac{1}{4}x^4-\tfrac{1}{3}x^3\right)\Big|_{-4}^{3} = \rho(-218.58\overline{3})$$

used calculator

$$M_x = \int_{-4}^{3}\frac{\rho}{2}\left[(16-x^2)^2-(x+4)^2\right]dx$$

$$= \frac{\rho}{2}\int_{-4}^{3}(256-32x^2+x^4-x^2-8x-16)\,dx$$

$$= \frac{\rho}{2}\int_{-4}^{3}(240-33x^2+x^4-8x)\,dx$$

$$= \frac{\rho}{2}\left(240x-11x^3+\tfrac{1}{5}x^5-4x^2\right]\Big|_{-4}^{3} = 480.2\rho$$