Part 1 Test 1 Summer II, 2012 Name: Key

MAT 162 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator and notes recorded on one side of a single 8.5 by 11 inch paper. Otherwise the test is closed book. When you turn in your test, staple your notes to Part 1.

For 1-4, calculate the following:

10 1. $\int x^{13} \ln(x)dx$ $= uv - \int v\,du$

$u = \ln x \quad dv = x^{13}dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{14}x^{14}$

$= \frac{1}{14}x^{14}\ln x - \int \frac{1}{14}x^{14}\frac{1}{x}dx$

$= \frac{1}{14}x^{14}\ln x - \frac{1}{14}\int x^{13}dx$

$= \frac{1}{14}x^{14}\ln x - \frac{1}{14}\frac{x^{14}}{14} + C = \boxed{\frac{1}{14}x^{14}\ln x - \frac{1}{14^2}x^{14} + C}$

10 2. $\int \cos^3(x)\sin^4(x)dx$ $= \int \cos^2 x \sin^4 x \cos x\,dx$

$= \int (1 - \sin^2 x)\sin^4 x \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int (1-u^2)u^4\,du$

$= \int (u^4 - u^6)du = \frac{1}{5}u^5 - \frac{1}{7}u^7 + C$

$= \boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$

10 3. $\int \frac{x^5}{\sqrt{16+x^2}}dx$ $= I$

Method I $x = 4\tan\theta$

$dx = 4\sec^2\theta\,d\theta$

$\sqrt{16+x^2} = \sqrt{16+16\tan^2\theta} = 4\sec\theta$

$\int \frac{4^5\tan^5\theta}{4\sec\theta}4\sec^2\theta\,d\theta$

$= 4^5\int \tan^5\theta\sec\theta\,d\theta$

$= 4^5\int (\tan^4\theta)\tan\theta\sec\theta\,d\theta$

$u = \sec\theta$

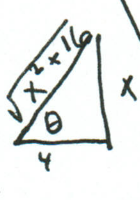
$du = \sec\theta\tan\theta\,d\theta$

$= 4^5\int (\sec^2\theta - 1)^2\tan\theta\sec\theta\,d\theta$

$= 4^5\int (u^2-1)^2du = 4^5\int (u^4 - 2u^2 + 1)du = 4^5\left[\frac{1}{5}u^5 - \frac{2}{3}u^3 + u\right] + C$

$= 4^5\left[\frac{1}{5}\sec^5\theta - \frac{2}{3}\sec^3\theta + \sec\theta\right] + C$

$4^5\left[\frac{1}{5}\left(\frac{\sqrt{x^2+16}}{4}\right)^5 - \frac{2}{3}\left(\frac{\sqrt{x^2+16}}{4}\right)^3 + \frac{\sqrt{x^2+16}}{4}\right] + C$



or

Method II $I = \frac{1}{2}\int \frac{(x^2)^2(2x)\,dx}{\sqrt{16+x^2}}$

$w = 16 + x^2$

$dw = 2x\,dx$

$= \frac{1}{2}\int \frac{(w-16)^2}{w^{\frac{1}{2}}}dw$

$= \frac{1}{2}\int (w^{3/2} - 32w^{1/2} - 16w^{-\frac{1}{2}})dw$

$= \frac{1}{2}\left[\frac{2}{5}w^{5/2} - 32\left(\frac{2}{3}\right)w^{3/2} - 32w^{\frac{1}{2}}\right]$

$= \frac{1}{2}\left[\left(\frac{2}{5}\right)(16+x^2)^{5/2} - \frac{64}{3}(16+x^2)^{3/2} - 32(x^2+16)^{\frac{1}{2}}\right] + C$

10 4. $\int \frac{7x+4}{x^2+10x-24}\,dx = \int \frac{7x+4}{(x+12)(x-2)}\,dx = \int \frac{A}{x+12} + \frac{B}{x-2}\,dx$

$7x+4 = A(x-2) + B(x+12)$

$x=2 \quad 18 = B(14) \Rightarrow B = \frac{9}{7}$

$x=-12 \quad 7(-12)+14 = A(-14)$

$A = \frac{7(12)-14}{14} = \frac{42}{7} - 1 = \frac{35}{7}$

$= \frac{40}{7}\int \frac{1}{x+12}\,dx + \frac{9}{7}\int \frac{1}{x-2}\,dx$

$= \frac{40}{7}\ln|x+12| + \frac{9}{7}\ln|x-2| + C$

8 5. Estimate $\int_5^{10} \cos(x^2-1)\,dx$ using Simpson's Rule with n = 6. Write the sum; you do not have to evaluate the sum.

$\Delta x = \frac{10-5}{6} = \frac{5}{6}$

where $f(x) = \cos(x^2-1)$

5, $\frac{35}{6}$, $\frac{40}{6}$, 7.5, $8\frac{1}{3}$, $9\frac{1}{6}$, 10

$\frac{\frac{5}{6}}{3}\left[f(5) + 4f\left(\frac{35}{6}\right) + 2f\left(\frac{40}{6}\right) + 4f(7.5) + 2f\left(8\frac{1}{3}\right) + 4f\left(9\frac{1}{6}\right) + f(10)\right]$

6. Calculate the following; it the integral does not converge, state "does not converge."

a. $\int_1^{+\infty} \frac{1}{x^2+4x+5}\,dx = \lim_{b\to+\infty} \int_1^b \frac{1}{(x+2)^2+1} = \lim_{b\to+\infty} \arctan(x+2)\Big|_1^b$

$= \lim_{b\to+\infty} \arctan(b+2) - \arctan 3 = \boxed{\frac{\pi}{2} - \arctan 3}$

9

b. $\int_{-3}^{3} \frac{x}{\sqrt{9-x^2}}\,dx = \lim_{b\to-3^+} \int_b^0 \frac{x}{\sqrt{9-x^2}}\,dx + \lim_{b\to3^-} \int_0^b \frac{x}{\sqrt{9-x^2}}\,dx$

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$= \lim_{b\to-3^+} -(9-x^2)^{\frac{1}{2}}\Big|_b^0 + \lim_{b\to3^-} -(9-x^2)^{\frac{1}{2}}\Big|_0^b$

$= \lim_{b\to-3^+} -3 + (9-b^2)^{\frac{1}{2}} + \lim_{b\to3^-} -(9-b^2)^{\frac{1}{2}} + 3$

$= -3 + 0 - 0 + 3 = 0$

7. Tell why the following converge or diverge:

9 a. $\int_1^{+\infty} \frac{2+(x\cos(x))^2}{x^4}\,dx$

$\frac{2+(x\cos x)^2}{x^4} \le \frac{2x^2+x^2}{x^4} = \frac{3}{x^2}$

$3\int_1^{+\infty} \frac{1}{x^2}$ converges

$\therefore \int_1^{+\infty} \frac{2+(x\cos x)^2}{x}$ converges

9 b. $\int_1^{+\infty} \frac{x^2+1}{2x^3-1}\,dx$

$\frac{1}{2}\frac{1}{x} \le \frac{x^2}{2x^3} \le \frac{x^2+1}{2x^3-1}$

$\int_1^{+\infty} \frac{1}{2}\frac{1}{x}\,dx$ diverges $\therefore \int_1^{+\infty} \frac{x^2+1}{2x^3-1}\,dx$ diverges

8. Calculate $\int \frac{2x+5}{x^2+8x+20}dx$ $= \int \frac{2x+5}{(x+4)^2+2^2}dx$

$= \int \frac{2(x+4)+5-8}{(x+4)^2+2^2}dx$

8 $= \int \frac{2(x+4)}{(x+4)^2+2^2}dx - 3\int \frac{1}{(x+4)^2+2^2}dx$

$= \boxed{\ln((x+4)^2+2^2) - \frac{3}{2}\arctan\left(\frac{x+4}{2}\right) + C}$

9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the integral): $\int \frac{4x+5}{(x^2+10x+26)^2(x^2+2x-15)^3}dx$

$(x^2+10x+26)^2 = \left((x+5)^2+1^2\right)^2$

$(x^2+2x-15)^3 = (x+5)^3(x-3)^3$

8 $\therefore$ $\frac{A_1x+B_1}{x^2+10x+26} + \frac{A_2x+B_2}{(x^2+10x+26)^2} + \frac{C_1}{x+5} + \frac{C_2}{(x+5)^2} + \frac{C_3}{(x+5)^3}$

$+ \frac{D_1}{x-3} + \frac{D_2}{(x-3)^2} + \frac{D_3}{(x-3)^3}$

Part II

(1) $\int \sqrt{\frac{1+x}{1-x}}\,dx = \int \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}}\,dx = \int \frac{1+x}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{x}{\sqrt{1-x^2}}\,dx$

$= \sin^{-1}x - \sqrt{1-x^2} + C$

(2) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+4\cot x}{4-\cot x}\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+\frac{4\cos x}{\sin x}}{4-\frac{\cos x}{\sin x}} \cdot \frac{\sin x}{\sin x}\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{4\cos x + \sin x}{4\sin x - \cos x}\,dx$

$u = 4\sin x - \cos x$
$du = (4\cos x + \sin x)\,dx$

$= \int_{\frac{3}{\sqrt{2}}}^{4} \frac{1}{u} = \ln|u|\Big|_{\frac{3}{\sqrt{2}}}^{4} = \ln 4 - \ln\frac{3}{\sqrt{2}} = \ln\left(\frac{4}{3}\sqrt{2}\right)$

(3) $\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x}} \cdot \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\,dx = \int\left(\sqrt{x+1}-\sqrt{x}\right)dx = \frac{2}{3}\left[(x+1)^{3/2} - x^{3/2}\right] + C$

(4) $\int \frac{\sqrt{x}}{1+x^3}\,dx$; $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$; $2u\,du = dx$

$\int \frac{u}{1+u^6}\,2u\,du = 2\int \frac{u^2}{1+(u^3)^2}\,du = \frac{2}{3}\int \frac{3u^2}{1+(u^3)^2}\,du = \frac{2}{3}\int \frac{1}{1+v^2}\,dv$

$v = u^3$
$dv = 3u^2\,du$

$= \frac{2}{3}\arctan v + C$

$\boxed{= \frac{2}{3}\arctan(u^3) + C}$

$\boxed{= \frac{2}{3}\arctan(x^{3/2}) + C}$

(5) $I = \int_0^\pi t\cos^2 t\,dt = \int_0^\pi \cos t \quad t\cos t\,dt$

$u = t\cos t$ $\quad dv = \cos t\,dt$
$du = (\cos t - t\sin t)\,dt$ $\quad v = \sin t$

$= t\cos t\sin t\Big|_0^\pi - \int_0^\pi \sin t(\cos t - t\sin t)\,dt$

$= 0 + \int_0^\pi \sin t(-\cos t)\,dt + \int_0^\pi \sin^2 t\,dt$

$= -\frac{1}{2}\sin^2 t\Big|_0^\pi + \int_0^\pi (1-\cos^2 t)\,dt$

$= 0 + \int_0^\pi t - I.$

$\therefore\ 2I = \int_0^\pi t\,dt = \frac{1}{2}t^2\Big|_0^\pi = \frac{\pi^2}{2} \Rightarrow \boxed{I = \frac{\pi^2}{4}}.$

(6) $I = \int x\sin^2 x\cos x\,dx$

$u = x$ $\quad dv = \sin^2 x\cos x\,dx$
$du = dx$ $\quad v = \frac{1}{3}\sin^3 x$

$= uv - \int v\,du = \frac{1}{3}x\sin^3 x - \int \frac{1}{3}\sin^3 x\,dx$

$= \frac{1}{3}x\sin^3 x + \frac{1}{3}\int(1-\cos^2 x)(-\sin x)\,dx$

$= \boxed{\frac{1}{3}x\sin^3 x + \frac{1}{3}\left(\cos x - \frac{1}{3}\cos^3 x\right) + C}$

(7) $\int \frac{x^3}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\int x^2 \frac{2x}{\sqrt{1+x^2}}\,dx$

$u = 1+x^2$
$du = 2x\,dx$

$= \frac{1}{2}\int \frac{u-1}{u^{\frac{1}{2}}}\,du = \frac{1}{2}\int\left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du = \frac{1}{2}\left[\frac{2}{3}u^{3/2} - 2u^{\frac{1}{2}}\right] + C$

$= \boxed{\frac{1}{3}(1+x^2)^{3/2} - (1+x^2)^{\frac{1}{2}} + C}$