

XIII. Tell why each series is conditionally convergent, absolutely convergent or divergent.

7

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3 + 18}$ is absolutely convergent since $\sum_{n=1}^{\infty} \left| \frac{(-1)^n n}{n^3 + 18} \right| = \sum_{n=1}^{\infty} \frac{n}{n^3 + 18}$

is convergent. (Compare $\frac{n}{n^3+18}$ to $\frac{1}{n^2}$: $\frac{\frac{n}{n^3+18}}{\frac{1}{n^2}} = \frac{n^3}{n^3+18} \rightarrow 1$.)

By limit comparison, $\sum_{n=1}^{\infty} \frac{n}{n^3+18}$ converges since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges because of p=2 series convergence)

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3 + 18}$ is conditionally convergent.

I. $\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 18}$ is divergent by limit comparison test and comparing to $\sum_{n=1}^{\infty} \frac{1}{n}$ which diverges ($p=1$).

II. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3 + 18}$ converges by Alt. Series (a) $\lim_{n \rightarrow \infty} \frac{n^3}{n^3 + 18} = 0$

$$\text{and (b)} \quad f(x) = \frac{x^2}{x^3 + 18} ; \quad f'(x) = \frac{(x^3 + 18)(2x) - x^2(3x^2)}{(x^3 + 18)^2} = \frac{36x - x^4}{(x^3 + 18)^2} < 0$$

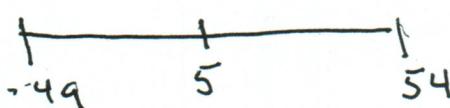
(c) $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^3 + 18}$ is divergent since $\frac{n^2}{n^3 + 18}$ is decreasing for $n \geq 1$ for x suff. large

Since $\lim_{n \rightarrow \infty} \frac{(-1)^n n^3}{n^3 + 18}$ is not zero.

XIV. Find the radius and interval of convergence for $f(x) = \sum_{n=1}^{\infty} (x-5)^n 7^{-2n} n^{-2}$

$$a_n = \frac{(x-5)^n}{7^{2n} n^2}; \quad \left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(x-5)^{n+1}}{7^{2(n+1)} (n+1)^2} \cdot \frac{7^{2n} n^2}{(x-5)^n} \right|$$

$$\therefore R = 49$$



$$= \frac{|x-5|}{7^2} \frac{n^2}{(n+1)^2} \rightarrow \frac{|x-5|}{7^2} < 1$$

(as $n \rightarrow \infty$)

check endpoints at $x = 54$, $\sum_{n=1}^{\infty} \frac{49^n}{7^{2n}} \frac{1}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges $P=2$ series
at $x = -49$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is absolutely convergent

\therefore Interval of convergence is $[-49, 54]$.

XV. Use a power series to estimate $\int_0^1 \frac{1}{1+x^5} dx$ with an error less than 10^{-12} .

$$\frac{1}{1+x^5} = \frac{1}{1-(-x^5)} = \sum_{n=0}^{\infty} (-x^5)^n = \sum_{n=0}^{\infty} (-1)^n x^{5n}$$

$$\int_0^1 \frac{1}{1+x^5} dx = \int_0^1 \sum_{n=0}^{\infty} (-1)^n x^{5n} dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \left[\frac{x^{5n+1}}{5n+1} \right]_0^1$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(-1)^{5n+1}}{5n+1}$$

$$n=2 \quad \left| \frac{(-1)^m \cancel{(-1)^{11}}}{11} \right| = \frac{10^{-12}}{1.1} < 10^{-12}$$

\therefore By Alt. Series

$$\boxed{\frac{1}{1} - \frac{(-1)^{10}}{6}}$$

has an error less than 10^{-12} ,