XIII. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1^n)n}{n^3+18}$ is absolutely convergent since $\sum_{n=1}^{\infty} \left| \frac{(-1)^n n}{n^3+18} \right| = \sum_{n=1}^{\infty} \frac{n}{n^3+18}$ is convergent. (Compare $\frac{n}{n^3+18}$ to $\frac{1}{n^2}$: $\frac{\frac{n}{n^3+18}}{\frac{1}{n^2}} = \frac{n^3}{n^3+18} \to 1$.

By Limit Comparison, $\sum_{n=1}^{\infty} \frac{n}{n^3+18}$ converges since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges because of $p=2$ series convergence)

(b) $\sum_{n=1}^{\infty} \frac{(-1^n)n^2}{n^3+18}$ is conditionally convergent.

I. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+18}$ is divergent by limit comparison test and comparing to $\sum_{n=1}^{\infty} \frac{1}{n}$ which diverges ($p=1$).

II. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+18}$ converges by Alt. Series: (a) $\lim_{n\to+\infty} \frac{n^2}{n^3+18} = 0$

and (b) $f(x) = \frac{x^2}{x^3+18}$; $f'(x) = \frac{(x^3+18)(2x) - x^2(3x^2)}{(x^3+18)^2} = \frac{36x - x^4}{(x^3+18)^2} < 0$

$\therefore \frac{n^2}{n^3+18}$ is decreasing for $n \geq 4$ for x suff. large

(c) $\sum_{n=1}^{\infty} \frac{(-1^n)n^3}{n^3+18}$ is divergent

since $\lim_{n\to+\infty} \frac{(-1)^n n^3}{n^3+18}$ is not zero.

XIV. Find the radius and interval of convergence for $f(x) = \sum_{n=1}^{\infty} (x-5)^n 7^{-2n} n^{-2}$.

$a_n = \frac{(x-5)^n}{7^{2n} n^2}$; $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x-5)^{n+1}}{7^{2(n+1)}(n+1)^2} \cdot \frac{7^{2n} n^2}{(x-5)^n}\right|$

$= \frac{|x-5|}{7^2} \frac{n^2}{(n+1)^2} \to \frac{|x-5|}{7^2} < 1$ as $n \to +\infty$.

$\therefore R = 49$

$-49 \qquad 5 \qquad 54$

check endpoints: at $x = 54$, $\sum_{n=1}^{\infty} \frac{49^n}{7^{2n}} \frac{1}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges $p = 2$ series

at $x = -49$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is absolutely convergent

$\therefore$ Interval of convergence is $[-49, 54]$.

XV. Use a power series to estimate $\int_0^{.1} \frac{1}{1+x^5}\,dx$ with an error less than 10^{-12}.

$$\frac{1}{1+x^5} = \frac{1}{1-(-x^5)} = \sum_{n=0}^{\infty} (-x^5)^n = \sum_{n=0}^{\infty} (-1)^n x^{5n}$$

$$\int_0^{.1} \frac{1}{1+x^5}\,dx = \int_0^{.1} \sum_{n=0}^{\infty} (-1)^n x^{5n}\,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{5n+1}}{5n+1}\Bigg|_0^{.1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(.1)^{5n+1}}{5n+1}$$

$n = 2$ $\left|\frac{(-1)^n (.1)^{11}}{11}\right| = \frac{10^{-12}}{1.1} < 10^{-12}$

$\therefore$ By Alt. Series

$\boxed{\frac{.1}{1} - \frac{(.1)^6}{6}}$ has an error less than 10^{-12}