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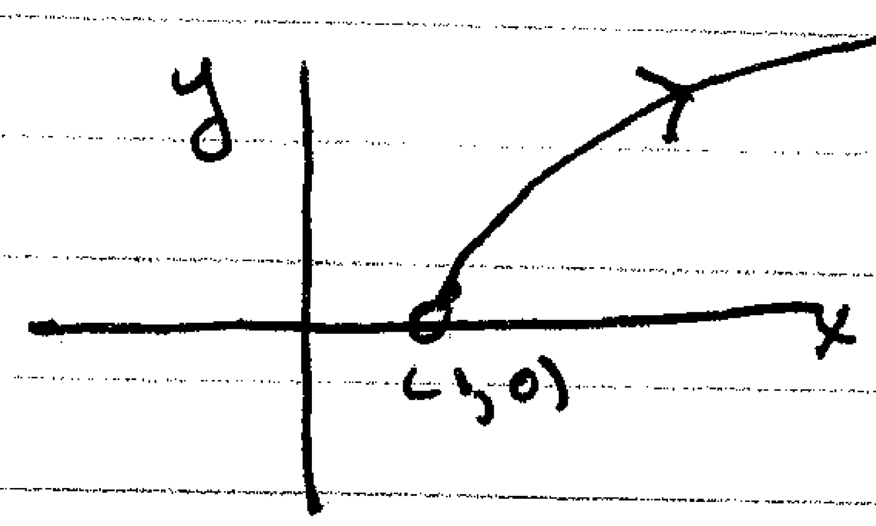
(2) $x = 1+e^{2t}$ $\quad y = e^t$ t

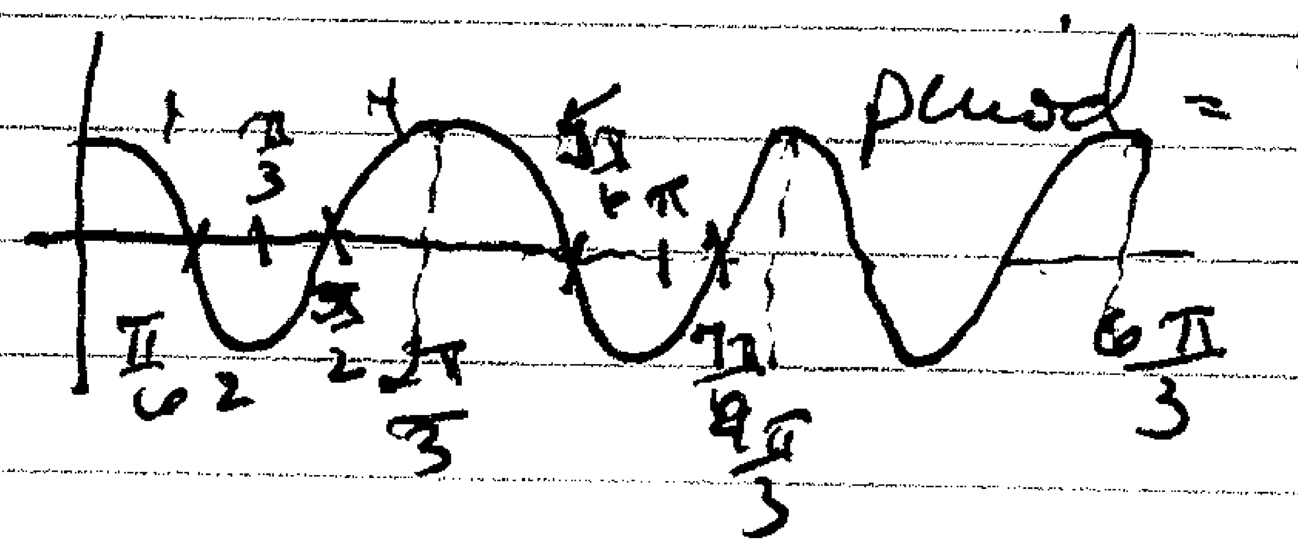
$x = 1+(e^t)^2 = 1+y^2$

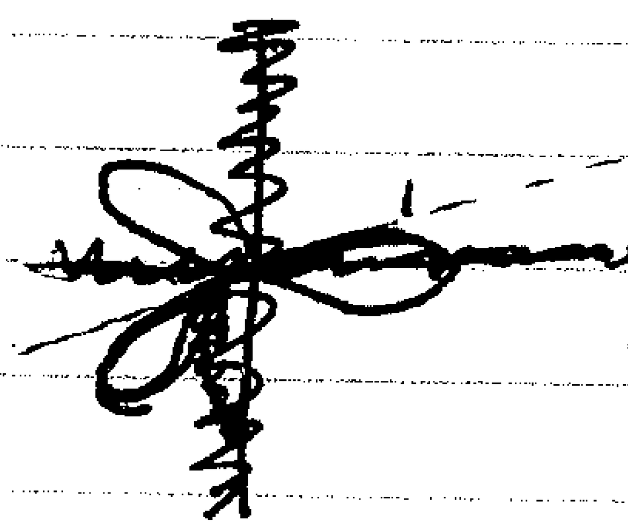
$x-1 = y^2 \quad y = +\sqrt{x-1}$



$-\infty < t < \infty$
$0 < e^t < \infty$
$0 < y < \infty$

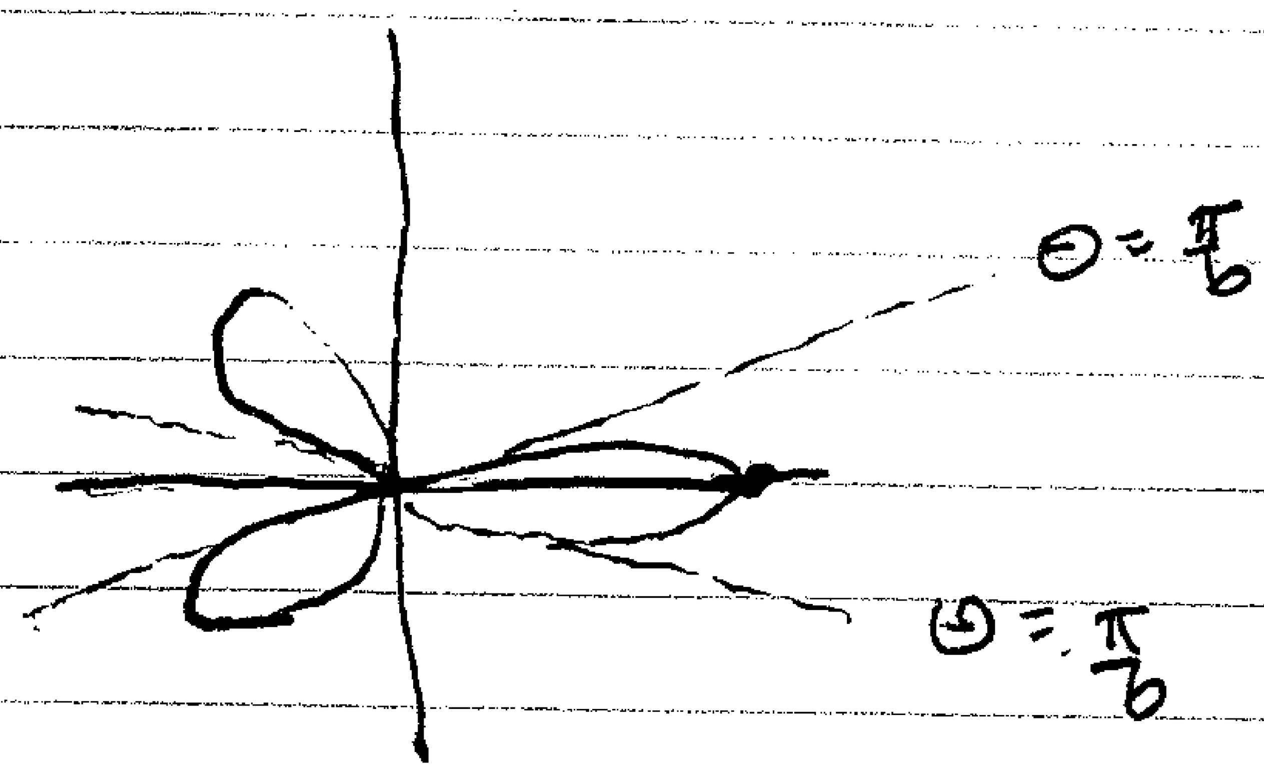
(11) period $= \frac{2\pi}{3}$



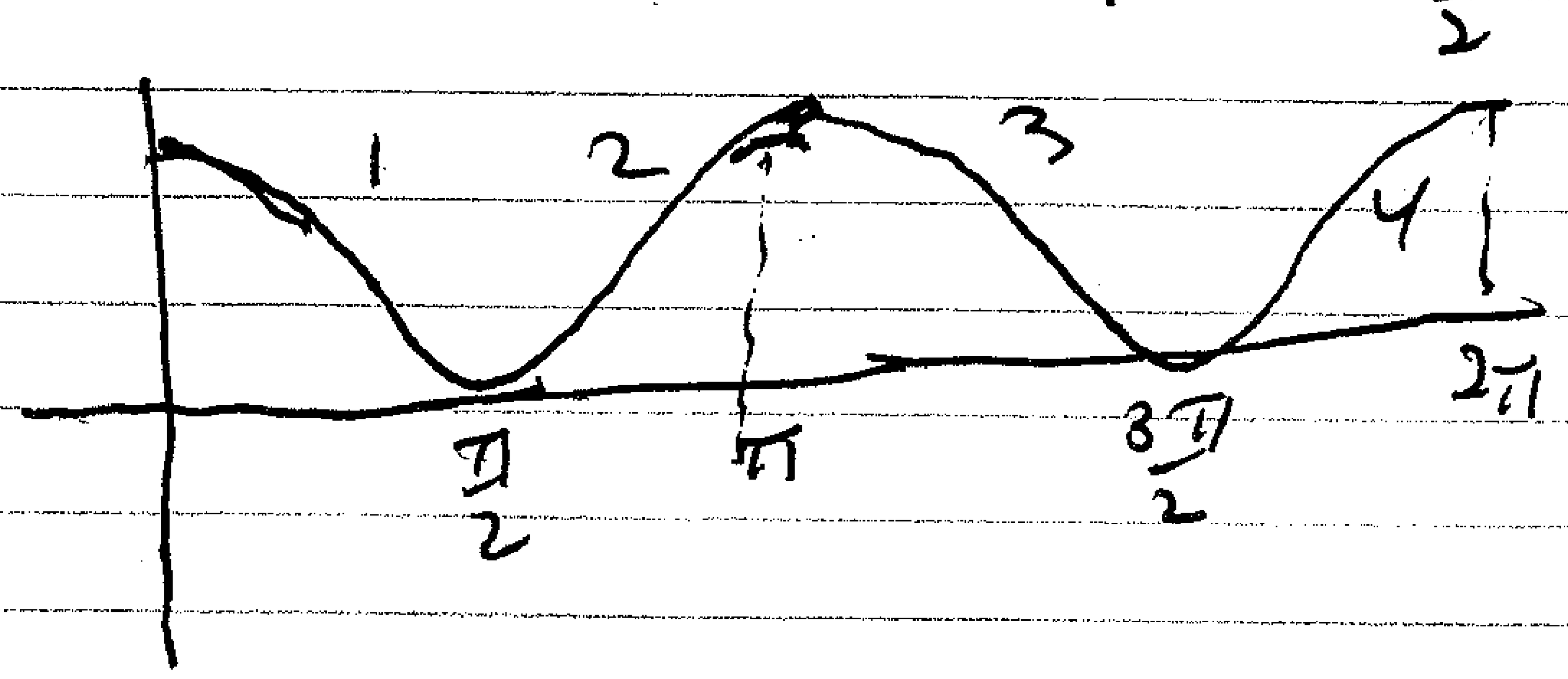


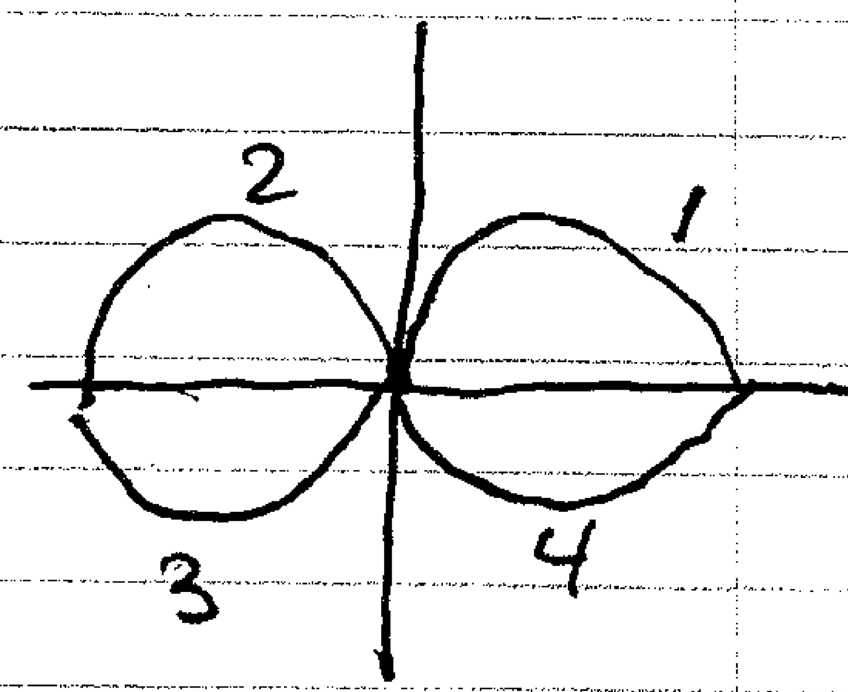
See next page for alt method

θ	r
0	1
$\frac{\pi}{6}$	0
$\pi/3$	-1
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	1
$5\pi/6$	0
π	-1
$7\pi/6$	0
$\frac{4}{3}\pi$	1

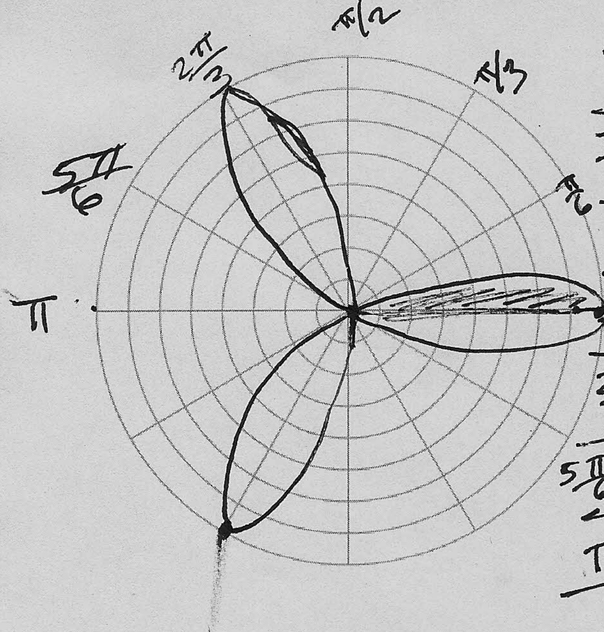


(13) $r = 1 + \cos 2\theta$ period $\frac{2\pi}{2} = \pi$

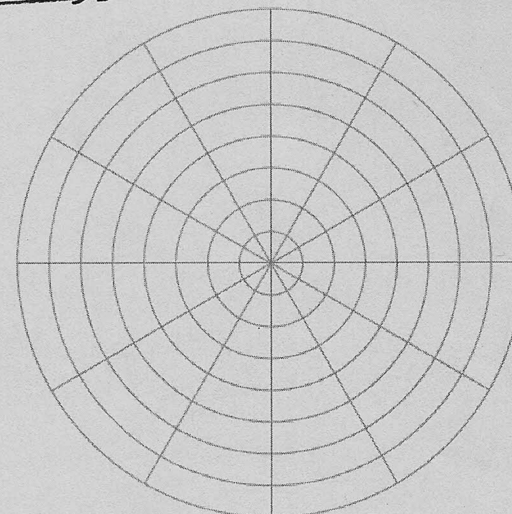




#11 $r = \cos 3\theta$



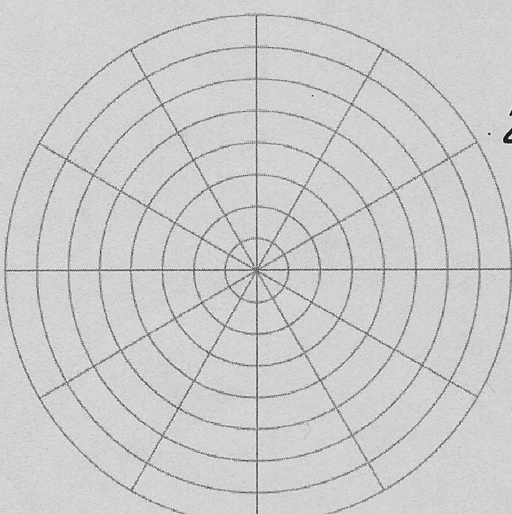
θ	3θ	$r = \cos(3\theta)$
0	0	1
$\frac{\pi}{6}$	$\frac{\pi}{2}$	0
$\frac{\pi}{3}$	π	-1
$\frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$\frac{2\pi}{3}$	2π	1
$\frac{5\pi}{6}$	$\frac{5\pi}{2}$	0
π	3π	-1

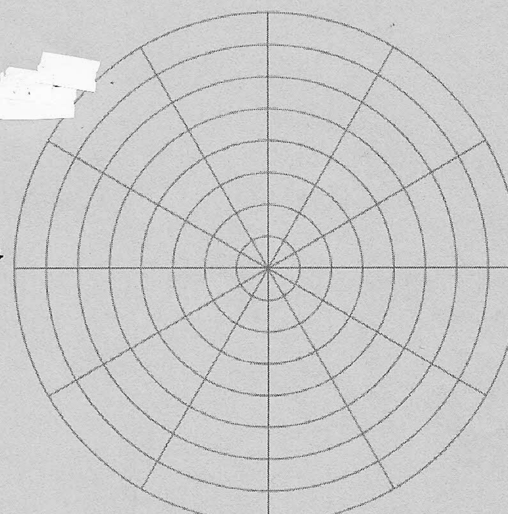


area of 1 leaf

$$2\int_0^{\pi/6} \left(\frac{1}{2} r^2\right) d\theta$$

$$\int_0^{\pi/6} \cos^2(3\theta)\, d\theta$$





(15) $r = \dfrac{3}{1+2\sin\theta}$

alt method -- keep it in polar form + do it that way.

$r + 2\boxed{\overset{y}{r\sin\theta}} = 3$

$r = 3 - 2r\sin\theta$

$r = 3 - 2y$ sq both sides

$x^2+y^2 = r^2 = (3-2y)^2$

$x^2+y^2 = 9 - 12y + 4y^2$

$x^2 - 3y^2 + 12y = 9$

$x^2 - 3(y^2 - 4y) = 9$

$x^2 - 3(y^2 - 4y + 4) = 9 - 12 = -3$

$$\boxed{(y-2)^2 - \frac{x^2}{3} = 1}$$

center $(0, \cancel{3}\,2)$

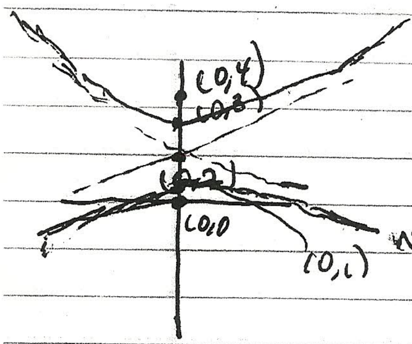
asymptotic lines $(y-2)^2 - \dfrac{x^2}{3} = 0$

$(y-2)^2 = \dfrac{x^2}{3}$

hyperbola

$y - 2 = \pm\dfrac{x}{\sqrt{3}}$

$\boxed{y = 2 \pm \dfrac{x}{\sqrt{3}}}$ $y = \frac{1}{\sqrt{3}}x + 2$ $y = -\frac{1}{\sqrt{3}}x + 2$



$c^2 = a^2 + b^2 = 3 + 1 \Rightarrow c = 2$

$\boxed{\text{foci} = (0, 2\pm 2) = \begin{cases}(0,0)\\(0,4)\end{cases}}$

vertices $(y-2)^2 = 1$

$y - 2 = \pm 1$

$\boxed{y = 2 \pm 1 = \begin{cases}3\\1\end{cases}}$

(25) $x = t + \sin t$; $y = t - \cos t$; $\dfrac{dx}{dt} = 1 + \cos t$; $\dfrac{dy}{dt} = 1 + \sin t$

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{1+\sin t}{1+\cos t}$ $0 = 1 + \sin t \Rightarrow \sin t = -1 \Rightarrow t = \dfrac{3\pi}{2} + 2k\pi$

horizontal tan lines at $\boxed{x = \frac{3\pi}{2} + 2k\pi - 1 \quad y = \frac{3\pi}{2} + 2k\pi}$

vertical tan lines when $1 + \cos t = 0$

$\cos t = -1$

$t = \pi + 2\pi k \Rightarrow$ $\boxed{x = 2\pi k + \pi \quad y = 2\pi k + \pi + 1}$

$\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{1+\sin t}{1+\cos t}\right)}{\frac{dx}{dt}} = \dfrac{(1+\cos t)\cos t - (1+\sin t)(-\sin t)}{(1+\cos t)^3} = \dfrac{\cos t + \sin t + 1}{(1+\cos t)^3}$

25) $x = t + \sin t \qquad y = t - \cos t$

find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$, V & H tangent line

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1+\sin t}{1+\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{dy/dx}{dx/dt} = \frac{\underline{(1+\cos t)(\cos t) - (1+\sin t)(-\sin t)}}{(1+\cos t)^3}$$

$$= \frac{\cos t + \cos^2 t + \sin t + \sin^2 t}{(1+\cos t)^3}$$

$$\frac{d^2y}{dx^2} = \frac{1+\cos t+\sin t}{(1+\cos t)^3}$$

vert: $\frac{1+\sin t}{1+\cos t} = \text{undef}$ $\therefore 1+\cos t = 0$

$\cos t = -1$

$t = \pi$

$x = t + \sin t$

$= \pi + \sin\pi = \pi$

$y = t - \cos t$

$= \pi - \cos\pi = \pi - (-1) = \pi + 1$

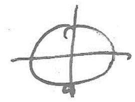
$(\pi, \pi+1)$ m undefined

$\boxed{x = \pi}$ vert tan line

horiz: $\frac{1+\sin t}{1+\cos t} = 0$

$\therefore 1 + \sin t = 0$

$\sin t = -1$



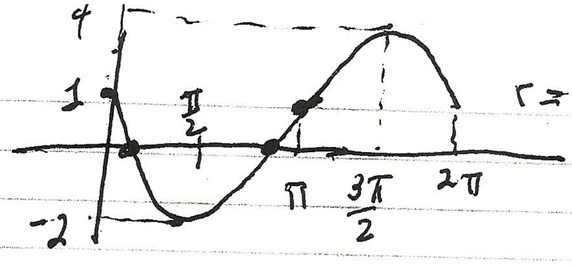
$t = 3\pi/2$

$y = t + \sin t$

$= \frac{3\pi}{2} - 1$

horiz tan line $y = 3\pi/2 - 1$

(32)



$r = 1 - 3\sin\theta$

$0 = 1 - 3\sin\theta$

$3\sin\theta = 1$

$\sin\theta = \frac{1}{3}$

$\theta = \sin^{-1}\left(\frac{1}{3}\right)$

and $\pi - \sin^{-1}\left(\frac{1}{3}\right)$

see next pg for picture

inner loop from $\sin^{-1}\left(\frac{1}{3}\right)$ to $\pi - \sin^{-1}\left(\frac{1}{3}\right)$

$$A = \int_{\sin^{-1}(\frac{1}{3})}^{\pi - \sin^{-1}(\frac{1}{3})} \frac{1}{2} r^2 \, d\theta = \frac{1}{2} \int_{\sin^{-1}(\frac{1}{3})}^{\pi - \sin^{-1}(\frac{1}{3})} (1 - 3\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2} \int_{\sin^{-1}(\frac{1}{3})}^{\pi - \sin^{-1}(\frac{1}{3})} (1 - 6\sin\theta + 9\sin^2\theta) \, d\theta$$

$$= \frac{1}{2} \int_{\sin^{-1}(\frac{1}{3})}^{\pi - \sin^{-1}(\frac{1}{3})} 1 - 6\sin\theta + 9\,\frac{1 - \cos 2\theta}{2} \, d\theta$$

etc.

$$= \frac{1}{2} \int_{\sin^{-1}(\frac{1}{3})}^{\pi - \sin^{-1}(\frac{1}{3})} \left(\frac{3}{2} - 6\sin\theta - \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{2}\left(\frac{3}{2}\theta + 6\cos\theta - \frac{\sin 2\theta}{4}\right)\Bigg|_{\sin^{-1}(\frac{1}{3})}^{\pi - \sin^{-1}(\frac{1}{3})}$$

you got it this far - OK

(37) $x = 3t^2 \quad y = 2t^3$

see later page for picture, etc.

$$L = \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt = \int_0^2 \sqrt{(6t)^2 + (6t^2)^2} \, dt$$

$$= \int_0^2 \sqrt{(6t)^2 (1 + t^2)} \, dt = \int_0^2 6t\sqrt{1 + t^2} \, dt$$

$u = 1 + t^2$

$\frac{du}{dt} = 2t$

$\frac{1}{2} du = t\,dt$

$3\,du = 6t\,dt$

$$= \int_1^5 u^{1/2} \, 3\,du$$

$$= 2u^{3/2}\Big|_1^5$$

$$= \boxed{2(5^{3/2} - 1)}$$

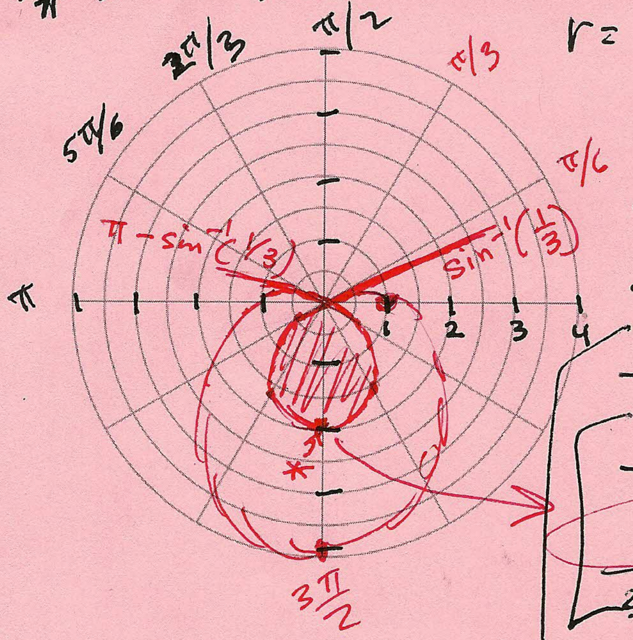
#32 find area inside small loop of

$r = 1 - 3\sin\theta \rightarrow r = 0$

$1 - 3\sin\theta = 0$

$1 = 3\sin\theta$

$\frac{1}{3} = \sin\theta$



θ	$r = 1 - 3\sin\theta$
0	$1 - 0 = 1$
$\frac{\pi}{6}$	$1 - 1.5 = -.5$
$\frac{\pi}{3}$	$1 - \frac{3\sqrt{3}}{2} \approx -1.6$
$\frac{\pi}{2}$	-2 *
$\frac{2\pi}{3}$	-1.6
$\frac{5\pi}{6}$	$-.5$
π	$1 - 0 = 1$
…	
$\frac{3\pi}{2}$	$1 - 3(-1) = 4$

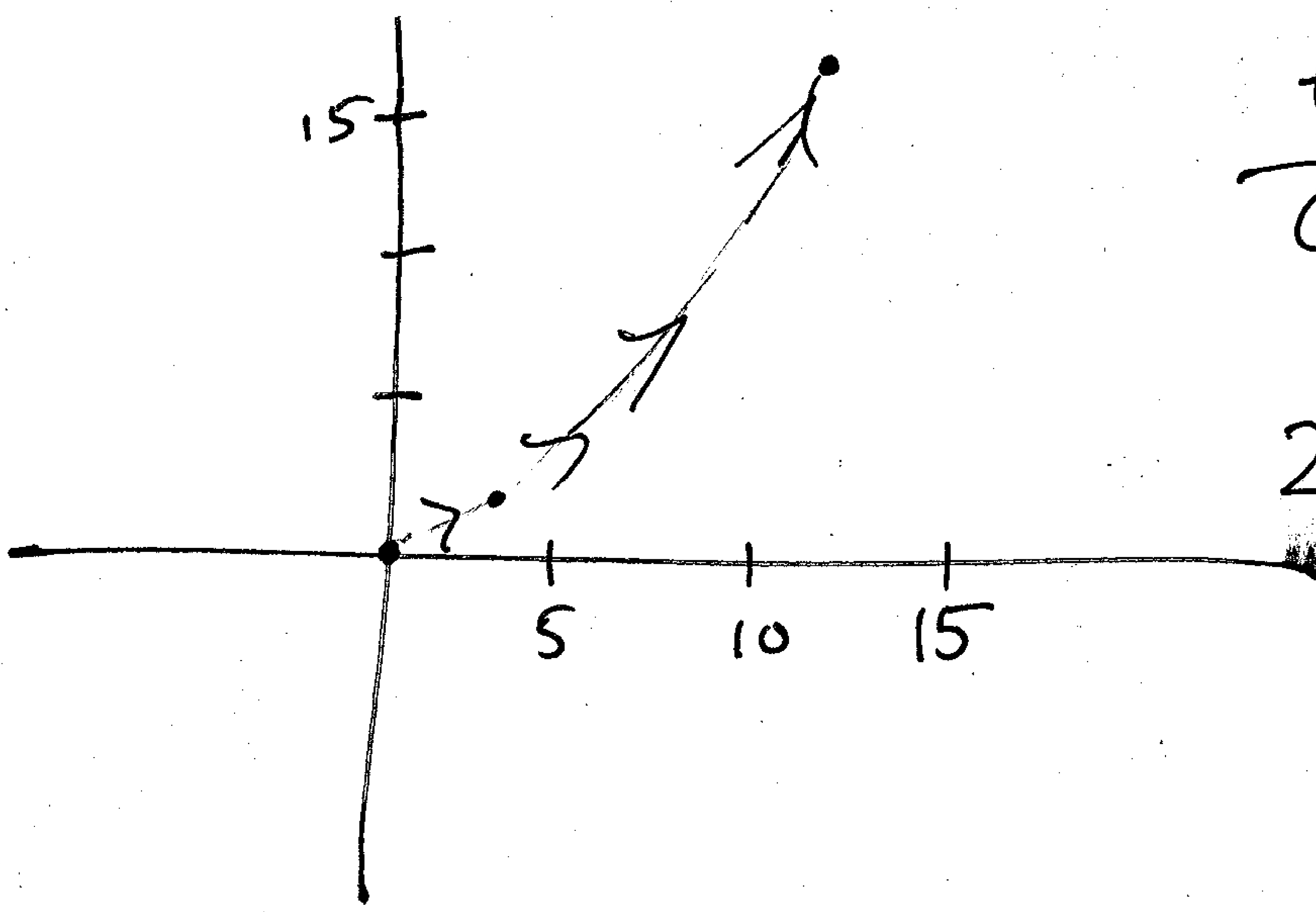
$$\int_{\sin^{-1}(\frac{1}{3})}^{\pi - \sin^{-1}(1/3)} \frac{1}{2} r^2 \, d\theta \quad \text{or}$$

$$2\int_{\sin^{-1}(\frac{1}{3})}^{\pi/2\,*} \frac{1}{2} r^2 \, d\theta$$

using half loop + symmetry

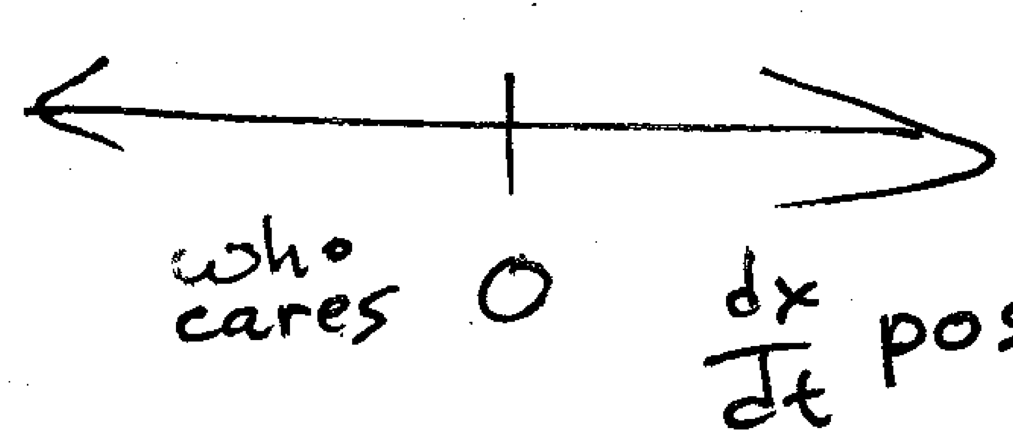
find arc length of $x = 3t^2,\ y = 2t^3$

$0 \le t \le 2$

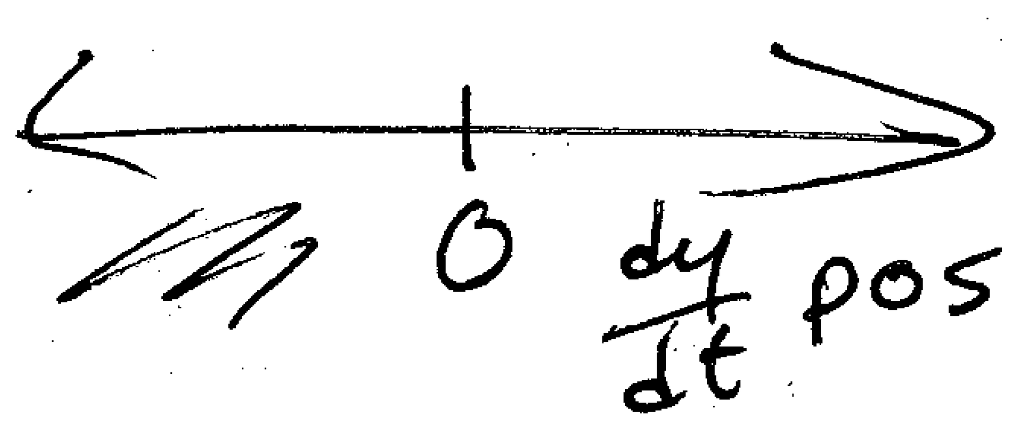


t	x	y
0	0	0
1	3	2
2	12	16

$\frac{dx}{dt} = 6t$



$\frac{dy}{dt} = 6t^2$



$$L = \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^2 \sqrt{36t^2 + 36t^4}\, dt$$

$$= \int_0^2 \underline{6t}\, \sqrt{1 + t^2}\, \underline{dt}$$

$u = 1 + t^2$ $\quad 0 \to 1$

$du = 2t\,dt$ $\quad 2 \to 5$

$$\int_1^5 3u^{1/2}\, du = \cancel{3}\tfrac{2}{\cancel{3}} u^{3/2} \Big|_1^5 = 2\left(5^{3/2} - 1\right)$$

$$= 2(1+t^2)^{3/2} \Big|_0^2 = 2(5^{3/2} - 1)$$

49 ellipse foci at $(\pm 4, 0)$ $\quad c = 4$

vertices at $(\pm 5, 0)$ $\quad a = 5$

$b^2 = a^2 - c^2 = 25 - 16 = 9 = 3^2 \Rightarrow b = 3$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$

52 foci $(3, \pm 2)$ $\quad c = 2$

Center $(3, 0)$

major axis length 8 $\quad \therefore a = 4$

$b^2 = 4^2 - 2^2 = 16 - 4 = 12$

$b = 2\sqrt{3}$

$b^2 = 12$

$$\boxed{\frac{(x-3)^2}{12} + \frac{y^2}{16} = 1}$$