MAT 162 Final Exam Name ____________________

Summer II 2011

For full credit, show all work.

I. Calculate the following”

a. $\int x\cos(15x)dx$

b. $\int x^3(x^2+9)^{3/2}dx$

II. Tell whether $\int_2^{+\infty}\frac{x^5}{x^6-2x-1}dx$ converges or diverges, and why.

III. Use the midpoint rule with n = 5 to estimate $\int_1^3 \frac{x^2}{1+x^4}\,dx$. [remember midpoint, trapezoid, Simpson's]

IV. Find the length of the graph of the curve $y = 2x^{1.5}, \quad 0 \le x \le 3$.

V. Find the centroid of the region bounded by the curves y = 2 - x , $y = x^2$, x = 0, and x = 1.

VI. Find k so that f(x) = $\frac{k}{x^2-1}$ if x≥2 and f(x) = 0 if x <2, is a probability density function.

VII. Solve completely:

(a) $\frac{dy}{dx} = \frac{4x}{(2x^2+1)y}$, y(0) = 2.

(b) $\frac{dy}{dx}$ - 2xy = $5e^{-x}$

(c) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$.

VIII. Use Euler's Method and a stepsize of h = 0.1 to estimate y(.2) where $\frac{dy}{dx} = .5x(1+y^2)$, y(0) = 2.

IX. A 2000 liter tank is initially filled with brine that contains 4 kg of dissolved salt. A salt solution of .003 kg/l enters the tank at a rate of 50 l/minute; the tank is continuously mixed and a solution drains from the tank at a rate of 50 l/minute. How much salt is in the tank at t minutes?

X. Find the foci and vertices and sketch the graph of $8y^2 + x^2 - 10x + 64y = 47$.

XI. Convert $r = 8\sin(\theta)$ into rectangular coordinates and sketch the graph. Find the slope of the tangent line at $\theta = \frac{\pi}{6}$.

XII. For $x = 12 - t^3$ and $y = t^3 - 27t$, $-4 < t < 4$

(a) Find the points where the parametric system has a vertical tangent line.

(b) Find the points where there are horizontal tangent lines.

(c) Find where x is increasing.

(d) Find where y is increasing.

(e) Sketch the graph of the system on an x-y coordinate system.

XIII. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1^n)n}{n^3+18}$

(b) $\sum_{n=1}^{\infty} \frac{(-1^n)n^2}{n^3+18}$

(c) $\sum_{n=1}^{\infty} \frac{(-1^n)n^3}{n^3+18}$

XIV. Find the radius and interval of convergence for f(x) = $\sum_{n=1}^{\infty}(x-5)^n 7^{-2n} n^{-2}$.

XV. Use a power series to estimate $\int_0^{.1} \frac{1}{1+x^5}\,dx$ with an error less than 10^{-12}.