MAT 162 Test 5—Version 1 Name Solutions
Fall 2017

For full credit, show all work.

1. Find the foci and vertices and sketch the graph of $x^2 - 8x + 4y^2 + 16y = 32$.

(20)

$x^2 - 8x + 16 + \underbrace{4(y^2+4y+4)}_{16} = 32 + 16 + 16 = 64$

$(x-4)^2 + 4(x+2)^2 = 64$ div thru by 64

$\frac{(x-4)^2}{64} + \frac{(x+2)^2}{16} = 1$

Ellipse

ctr $= (4,-2)$

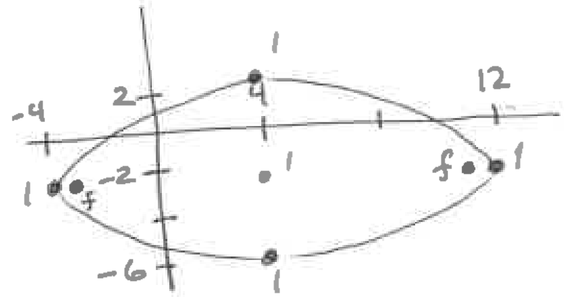
vertices: L+R 8, up+down 4

$(4\pm 8, -2) \to (-4,-2)$ + $(12,-2)$

$(4, -2\pm 4) \to (4,2)$ + $(4,-6)$

foci: $c^2 = 64 - 16 = 48$

L+R $\sqrt{48} = 4\sqrt{3} \approx 7$ $(4 \pm 4\sqrt{3}, -2)$



2. Find the foci and vertices and sketch the graph of $x^2 - 8x + 16y = 32$.

(20)

$16y - 32 = -x^2 + 8x$

$16y - 32 = -(x^2 - 8x)$

$16y - \underbrace{32 - 16}_{-48} = -(x^2 - 8x + 16)$ ← -16

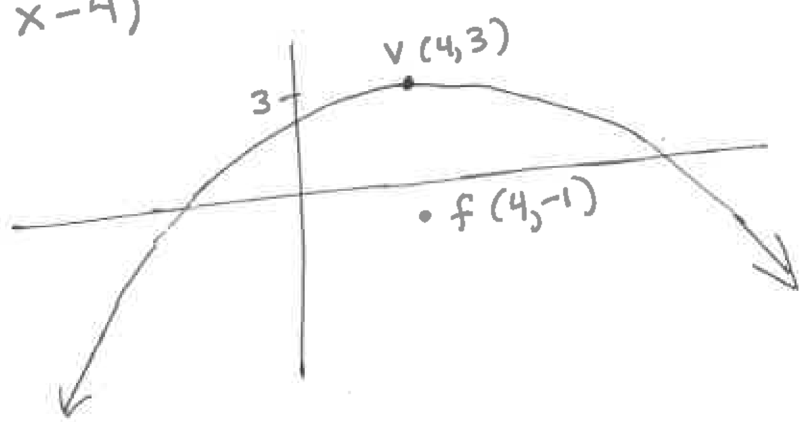
$16(y-3) = -(x-4)^2$ vert $= (4,3)$

$y - 3 = -\frac{1}{16}(x-4)^2$ focus $= (4,-1)$

$y - 3 = \frac{1}{4(-4)}(x-4)^2$

parabola...

opens down...



3. Convert r = 3/(1-2cos(θ)) into rectangular coordinates and sketch the graph. State the foci and vertices. Find the slope of the tangent line at $\theta = \frac{\pi}{2}$.

$r - 2r\cos\theta = 3$

$r = 3 + 2r\cos\theta$

2 $r^2 = (3 + 2r\cos\theta)^2$

$x^2 + y^2 = (3 + 2x)^2 = 9 + 12x + 4x^2$

2 $-3x^2 - 12x + y^2 = 9$

$-3(x^2 + 4x) + y^2 = 9$

$-3(x^2 + 4x + 4) + y^2 = 9 - 12 = -3$

2 $\frac{(x+2)^2}{1} - \frac{y^2}{3} = 1$

6

$c^2 = 1 + 3 = 4$

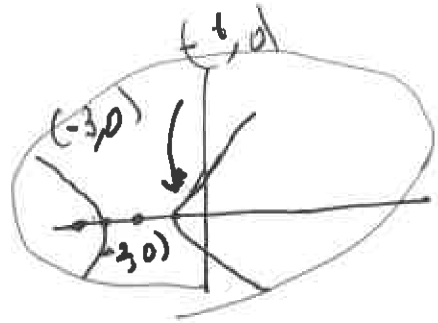
Center $(-2, 0)$

vertices $(-2 \pm 1, 0)$

$(-3, 0)$ $(-1, 0)$ 2

foci $(-2 \pm 2, 0)$

$(-4, 0)$ $(0, 0)$ 2



4

$x = r\cos\theta$ $\cos\frac{\pi}{2} = 0$ 1

$y = r\sin\theta$ $\sin\frac{\pi}{2} = 1$ 1

$r = \frac{3}{1 - 2\cos\theta}$ $r = \frac{3}{1} = 3$ 1

$\frac{dr}{d\theta} = \frac{-3}{(1 - 2\cos\theta)^2}(+2\sin\theta)$ $x = 3 \cdot 0 = 0$

$y = 3 \cdot 1 = 3$ 1

$\frac{dr}{d\theta} = \frac{-3(2)}{1} = -6$

$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$ 1

$= \frac{(-6)(1) + 3(0)}{-6(0) - 3(1)} = \boxed{2}$ 1 $y - 3 = 2(x - 0)$

$\boxed{y = 2x + 3}$

4. For $x = t^3 - 3t$ and $y = t^3 - 12t$, $-4 \le t \le 4$

(a) Find the x and y coordinates of the points where the parametric system has a vertical tangent line.

$\frac{dx}{dt} = 3t^2 - 3 = 3(t^2-1) = 3(t-1)(t+1)$ 2

t	x	y
-1	2	11
1	-2	-11

(b) Find the x and y coordinates of the points where there are horizontal tangent lines.

$\frac{dy}{dt} = 3t^2 - 12 = 3(t^2-4) = 3(t-2)(t+2)$ 2

t	x	y
-2	-2	16
2	2	-16

(c) Find the intervals of t where x is increasing.

t+1: − + +
t−1: − − +
+ −1 − 1 +

x is inc on $(-\infty, -1) \cup (1, +\infty)$

(d) Find the intervals of t where y is increasing.

t+2: − + +
t−2: − − +
+ −2 − 2 +

y inc on $(-\infty, -2) \cup (2, +\infty)$

(e) Find the equation of the tangent line when t = 0.

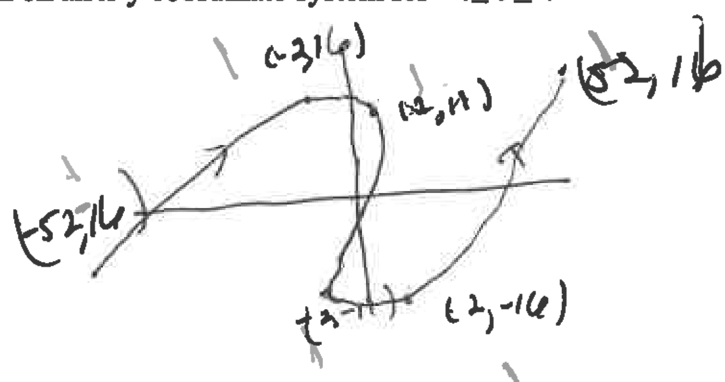
$\frac{dy}{dx}\Big|_{t=0} = \frac{-12}{-3} = 4$

x = 0
y = 0

$y - 0 = 4(x-0)$
$y = 4x$

(f) Sketch the graph of the system on an x-y coordinate system for $-4 \le t \le 4$

t	x	y
-4	-52	-16
-2	-2	16
-1	2	11
0	0	0
1	-2	-11
2	2	-16
4	52	16



MAT 162 Test 5—Version 2 Name Solutions

Fall 2017

For full credit, show all work.

(20) 1. Find the foci and vertices and sketch the graph of $x^2 + 8x + 4y^2 - 16y - 32 = 0$.

$$x^2+8x+16+\underbrace{4(y^2-4y+4)}_{+16} = 32+16+16$$

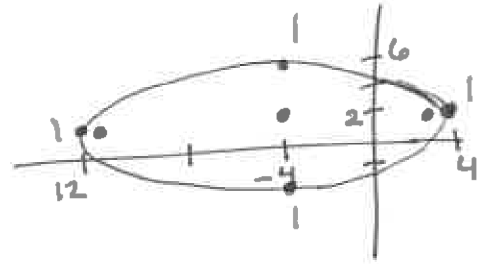
$$(x+4)^2+4(y-2)^2 = 64 \quad \text{Divide through by 64.}$$

$$\frac{(x+4)^2}{64}+\frac{(y-2)^2}{16} = 1 \quad \text{ellipse}$$

center $= (-4, 2)$

vertices: L & R 8 units from ctr
$(-4\pm 8, 2) \to (-12, 2)$ & $(4, 2)$
up & down 4 units from ctr
$(-4, 2\pm 4) \to (-4, -2)$ & $(-4, 6)$

foci: $c^2 = 64-16 = 48$, so $c = \sqrt{48} = 4\sqrt{3} \approx 7$
$(-4\pm 4\sqrt{3}, 2)$



2. Find the foci and vertices and sketch the graph of $x - 8y + 16y^2 = 32$.

(20)

$$16y^2 - 8y = 32 - x$$

$$16\left(y^2 - \tfrac{1}{2}y\right) = 32 - x$$

$$\underbrace{16\left(y^2 - \tfrac{1}{2}y + \tfrac{1}{16}\right)}_{+1} = 32 - x + 1$$

$$16\left(y-\tfrac{1}{4}\right)^2 = 33 - x = -(x-33)$$

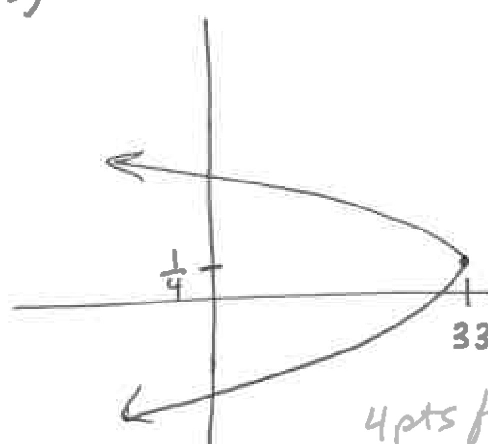
$$-16\left(y-\tfrac{1}{4}\right)^2 = (x-33)$$

$$\frac{1}{4\left(-\frac{1}{64}\right)}\left(y-\tfrac{1}{4}\right)^2 = x - 33$$

vertex $= \left(33, \tfrac{1}{4}\right)$

$p = -\frac{1}{64}$

focus $= \left(33 - \frac{1}{64}, \frac{1}{4}\right)$



4 pts for parabola + direction

3. Convert r = 3/(1+2cos(θ)) into rectangular coordinates and sketch the graph. State the foci and vertices.

(20) Find the slope of the tangent line at $\theta = \frac{\pi}{2}$.

$r + 2r\cos\theta = 3$

$r = 3 - 2r\cos\theta$

2 $r^2 = (3 - 2r\cos\theta)^2$

2 $x^2 + y^2 = (3-2x)^2 = 9 - 12x + 4x^2$

$-3x^2 + 12x + y^2 = 9$

$-3(x^2 - 4x) + y^2 = 9$

$-3(x^2 - 4x + 4) + y^2 = 9 - 12 = -3$

2 $+\frac{(x-2)^2}{1} - \frac{y^2}{3} = 1$

6

center $(2, 0)$

vertices $(2 \pm 1, 0)$ 2

$(3, 0)$ and $(1, 0)$

$c^2 = a^2 + b^2 = 1 + 3 = 4$

$c = 2$

foci $(2 \pm 2, 0)$ 2

$(0,0)$ + $(4,0)$

at $\theta = \frac{\pi}{2}$

$x = 3 \cdot 0 = 0$

$y = 3(1) = 3$

4

$x = r\cos\theta$

$y = r\sin\theta$

$r = \frac{3}{1+2\cos\theta}$

$\frac{dr}{d\theta} = -\frac{-3(2\sin\theta)}{(1+2\cos\theta)^2}$

$r = \frac{3}{1} = 3$ 2

$\frac{dr}{d\theta} = \frac{-3(2)}{1} = +6$ 2

$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

$\frac{+6(1) + 3(0)}{+6(0) - 3(1)} = \boxed{-2}$ 1

4

-2 is the answer

4. For $x = t^3 - 12t$ and $y = t^3 - 3t$, $-4 \le t \le 4$

(40) (a) Find the x and y coordinates of the points where the parametric system has a vertical tangent line.

$$\frac{dx}{dt} = 3t^2 - 12 = 3(t^2 - 4) = 3(t-2)(t+2) \quad 2$$

6

t	x	y
-2	16	-2
2	-16	2

(b) Find the x and y coordinates of the points where there are horizontal tangent lines.

$$\frac{dy}{dt} = 3t^2 - 3 = 3(t-1)(t+1) \quad 2$$

6

t	x	y
-1	11	2
1	-11	-2

(c) Find the intervals of t where x is increasing.

6

t+2   −   +   +

t−2   −   −   +

\+   −2   −   2   +

x is inc on $(-\infty, -2) \cup (2, +\infty)$ 2   2

(d) Find the intervals of t where y is increasing.

6

t+1   −   +   +

t−1   −   −   +

\+   −1   −   1   +

y is inc on $(-\infty, -1) \cup (1, +\infty)$ 2   2

(e) Find the equation of the tangent line when $t = 0$.

8

$$\frac{dy}{dx} = \frac{-3}{-12} = \frac{1}{4} \quad 4$$

$x = 0$, $y = 0$ 2

$$y - 0 = \frac{1}{4}(x - 0)$$
$$y = \frac{1}{4}x \quad 2$$

(f) Sketch the graph of the system on an x-y coordinate system for $-4 \le t \le 4$

8

t	x	y	
-4	-16	-52	
-2	16	-2	vertical
-1	11	2	horizontal
0	0	0	
1	-11	-2	horizontal
2	-16	2	vertical
4	16	52