I have graded question 4 — the tough one. Grade by process — if the process is OK ~~after~~ the mistake give credit for the correct process.

Test 3 Fall, 2017 Name: Key
MAT 162 version 1 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. The test is closed book.

1. Use Euler's Method to approximate y(1.1) given $\frac{dy}{dx} = 10x + 5y$, and y(1) = 2 Use a stepsize of 0.05 .

x	y	$(10x+5y)(.05)$
1	2	$(20)(.05) = 1$
1.05	3	$(10.5+15)(.05) = (25.5)(.05) = 1.275$
1.1	4.275	

16

2 points per underline

$\boxed{\text{Answer } 4.275}$

2. Find y(x), the solution to $\frac{dy}{dx} = (2+y)(x^2+x) \cdot y(0) = \pi/4$.

(15)

3 $\int \frac{1}{2+y}\,dy = \int (x^2+x)\,dx$

3 $\ln(2+y) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + C$

3 $2+y = e^{\frac{1}{3}x^3+\frac{1}{2}x^2} K$ where $e^c = K$

3 $y = -2 + Ke^{\frac{1}{3}x^3+\frac{1}{2}x^2}$

2 $\frac{\pi}{4} = y(0) = -2 + Ke^0 \Rightarrow K = 2 + \frac{\pi}{4} = 2.785$

1 $\therefore \boxed{y(x) = -2 + (2+\frac{\pi}{4})e^{\frac{1}{3}x^3+\frac{1}{2}x^2}}$

3. Find y(x), the solution to $\frac{dy}{dx} = 15x + \frac{2y}{x} \cdot y(1) = 2$.

(15)

3 $\frac{dy}{dx} - \frac{2}{x}y = 15x$

3 $I(x) = e^{-\int \frac{2}{x}dx} = e^{-2\ln x} = x^{-2}$

3 $\frac{d}{dx}(yx^{-2}) = 15x(x^{-2}) = \frac{15}{x}$

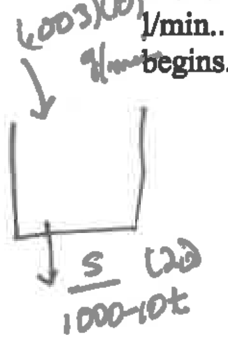
3 $yx^{-2} = \int \frac{15}{x}\,dx = 15\ln x + c$

1 $y = x^2(15\ln x + c)$

1 $2 = y(1) = 1(15\ln 1 + c) = c$

1 $\boxed{y = x^2(15\ln x + 2)}$

4. A large tank is filled with 1000 liters of contaminated water with a concentration of .005 grams of toxins per liter of water. Water containing .003 g of toxin per liter is pumped in at a rate of 10 l/min., mixes instantaneously, and then is pumped out at a rate of 20 l/min.. Find y(t) the number of grams of the toxin in the tank t minutes after the rinse begins. Then calculate y(50).



2 $S(t)$ = amount of toxin in tank at time t

2 $S(0) = 1000(.005) = 5$ grams

4 $\frac{dS}{dt}$ = rate in − rate out $= .03 - \frac{1}{50-\frac{1}{2}t} S$

10 2 $\frac{dS}{dt} + \frac{1}{50-\frac{1}{2}t} S = .03, \quad S(0)=5$

(25)

3 $I(t) = e^{\int \frac{1}{50-\frac{1}{2}t}dt} = e^{-2\ln(50-\frac{1}{2}t)} = (50-\frac{1}{2}t)^{-2}$

3 $\frac{d}{dt}\left(S(50-\frac{1}{2}t)^{-2}\right) = .03(50-\frac{1}{2}t)^{-2}$

3 $S(50-\frac{1}{2}t)^{-2} = \int .03(50-\frac{1}{2}t)^{-2}dt = .03(2)(50-\frac{1}{2}t)^{-1} + C$

3 $S = .06(50-\frac{1}{2}t) + C(50-\frac{1}{2}t)^2$

1 $\begin{cases} 5 = S(0) = .06(50) + C(50)^2 \\ \frac{2}{50^2} = C \end{cases} \Rightarrow$ 1 $S(t) = .06(50-\frac{1}{2}t) + \frac{2}{50^2}(50-\frac{1}{2}t)^{2}$

1 $S(50) = .06(25) + \frac{2}{50^2}(25)^2 = 1.5 + \frac{2}{4} = \boxed{\cancel{2}} = 2.00\ g$

5. First find the solution to $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} - 24y = 0$, $y(0) = 1$, $y'(0) = 2$.

(15)

1) $r^2 + 10r - 24$

1) $(r+12)(r-2) = 0$

1) $r = -12, \ r = 2$

4) $y(x) = c_1 e^{-12x} + c_2 e^{2x}$

2) $y'(x) = -12c_1 e^{-12x} + 2c_2 e^{2x}$

1) $\begin{cases} 1 = y(0) = c_1 + c_2 \\ 2 = y'(0) = -12c_1 + 2c_2 \end{cases}$
1)

3) $\begin{cases} 12 = 12c_1 + 12c_2 \\ 2 = -12c_1 + 2c_2 \\ \hline 14 = 14c_2 \\ c_2 = 1 \\ c_1 = 1 - c_2 = 0 \end{cases}$

$\boxed{y(x) = e^{2x}}$

6. A biological population is growing at a rate directly proportional to the size of the population. At $t = 0$ hours, the population is 100 units and at $t = 2$ hours the population is 150 units. Find the population at $t = 5$ hours.

$$\frac{dP}{dt} = kP, \quad P(0) = 100$$

(14)

5 (5) $P(t) = 100e^{kt}$

(2) $150 = P(2) = 100e^{2k}$

(4) $1.5 = e^{2k}$

$\ln 1.5 = 2k$

$\frac{1}{2}\ln 1.5 = k$

(2) $P(t) = 100\, e^{\frac{t}{2}\ln 1.5}$

(1) $\underbrace{P(5) = 100\, e^{\frac{5}{2}\ln 1.5} = 100(1.5)^{5/2} = 275.57}_{\text{any are acceptable}}$

Test 3 Fall, 2017 Name: Key

MAT 162 version 2 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. The test is closed book.

1. Use Euler's Method to approximate y(2.1) given $\frac{dy}{dx} = x + 10y$, and y(2) = 1 Use a stepsize of 0.05 .

(16)

x	y	(x + 10y)(.05)
2	1	(12)(.05) = .6
2.05	1.6	(2.05 + 16)(.05) = .9025
2.1	2.5025	

2 points per underline

$\boxed{\text{Answer } 2.5025}$

2. Find y(x), the solution to $\frac{dy}{dx} = (2 + y)(x^2 + x^3) \cdot y(0) = 1$.

(15)

(3) $\int \frac{1}{2+y}\,dy = \int (x^2+x^3)\,dx$

(3) $\ln(2+y) = \frac{1}{3}x^3 + \frac{1}{4}x^4 + c$

(3) $2+y = e^{\frac{1}{3}x^3 + \frac{1}{4}x^4} K$ where $e^c = K$

(3) $y = -2 + Ke^{\frac{1}{3}x^3 + \frac{1}{4}x^4}$

(3) $1 = y(0) = -2 + K \Rightarrow K = 3$

$\therefore \boxed{y(x) = -2 + 3e^{\frac{1}{3}x^3 + \frac{1}{4}x^4}}$

3. Find y(x), the solution to $\frac{dy}{dx} = 15x + \frac{2y}{x} \cdot y(2) = 1$.

(15)

(3) $\frac{dy}{dx} - \frac{2}{x}y = 15x$

(3) $I(x) = e^{-\int \frac{2}{x}dx} = e^{-2\ln x} = x^{-2}$

(3) $\frac{d}{dx}(yx^{-2}) = 15x(x^{-2}) = \frac{15}{x}$

(3) $yx^{-2} = \int \frac{15}{x}dx = 15\ln x + c$

(1) $y = x^2(15\ln x + c)$

(1) $1 = y(2) = 4(15\ln 2 + c) \Rightarrow c = \frac{1}{4} - 15\ln 2 =$

(1) $y = x^2(15\ln x + \frac{1}{4} - 15\ln 2)$

$.002(10)$ g/m

4. A large tank is filled with 1000 liters of contaminated water with a concentration of .005 grams of toxins per liter of water. Water containing .002 g of toxin per liter is pumped in at a rate of 10 l/min., mixes instantaneously, and then is pumped out at a rate of 20 l/min.. Find y(t) the number of grams of the toxin in the tank t minutes after the rinse begins. Then calculate y(50).

2 $S(t)$ = amount of toxin in tank at time t

2 $S(0) = 1000(.005) = 5$ grams

$\frac{S(20)}{1000-10t}$

4 $\frac{dS}{dt}$ = rate in − rate out $= .02 - \frac{1}{50-\frac{1}{2}t} S$

2 $\frac{dS}{dt} + \frac{1}{50-\frac{1}{2}t} S = .02$, $S(0)=5$

3 $I(t) = e^{\int \frac{2}{50-\frac{1}{2}t} dt} = e^{-2\ln(50-\frac{1}{2}t)} = (50-\frac{1}{2}t)^{-2}$

3 $\frac{d}{dt}\left(S(50-\frac{1}{2}t)^{-2}\right) = .02(50-\frac{1}{2}t)^{-2}$

(25) 3 $S(50-\frac{1}{2}t)^{-2} = \int .02(50-\frac{1}{2}t)^{-2} dt = (.02)(2)(50-\frac{1}{2}t)^{-1} + C$

3 $S = .04(50-\frac{1}{2}t) + C(50-\frac{1}{2}t)^2$

1 $5 = S(0) = .04(50) + C(50^2) \Rightarrow C = \frac{3}{50^2}$

1 $S(t) = .04(50-\frac{1}{2}t) + \frac{3}{50^2}(50-\frac{1}{2}t)^2$

1 $S(50) = .04(25) + \frac{3}{50^2}(25^2) = 1 + \frac{3}{4} = 1.75$ g

5. First find the solution to $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$, $y(0) = 1$, $y'(0) = 2$.

$r^2 + 10r + 24 = 0$

$(r+6)(r+4) = 0$

$r = -6 \quad r = -4$

(15)

$y(x) = c_1 e^{-6x} + c_2 e^{-4x}$

$y'(x) = -6c_1 e^{-6x} - 4c_2 e^{-4x}$

$$\begin{cases} 1 = y(0) = c_1 + c_2 \\ 2 = y'(0) = -6c_1 - 4c_2 \end{cases}$$

$6 = 6c_1 + 6c_2$

$2 = -6c_1 - 4c_2$

$8 = 2c_2$

$c_2 = 4$

$c_1 = 1 - c_2 = 1 - 4 = -3$

$\boxed{y(x) = -3e^{-6x} + 4e^{-4x}}$

6. A biological population is growing at a rate directly proportional to the size of the population. At t = 0 hours, the population is 100 units and at t= 2 hours the population is 250 units. Find the population at t = 3 hours.

(14) $\frac{dP}{dt} = kP, \quad P(0) = 100$

5 $P(t) = 100e^{kt}$

2 $250 = P(2) = 100e^{2k}$

.4 $2.5 = e^{2k}$

$\ln 2.5 = 2k$

$\frac{1}{2}\ln 2.5 = k$

2 $P(t) = 100\, e^{\frac{t}{2}\ln 2.5}$

1 $P(3) = \underbrace{100\, e^{3/2 \ln 2.5} = 100(2.5)^{3/2} = 395.28}_{\text{any are acceptable}}$