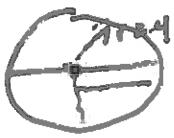
Fall, 2017 | Name: Key
MAT 162001 | Test 2 Version 1 | Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book. Each problem is worth 20 points. Problem 6 is 20 points extra credit.

1. A cylindrical tank 8 feet in diameter is lying on its side and is half full of oil of density 74 lbs/ft^3 . What is the hydrostatic force on one end of the tank?



$x^2+y^2=4^2$

$x=\sqrt{16-y^2}$

$w(y)=2x$

$=2\sqrt{16-y^2}$

depth $=-y$

$$\int_{-4}^{0} 74(-y)\,2\sqrt{16-y^2}\,dy = 74\left(\tfrac{2}{3}\right)(16-y^2)^{3/2}\Big|_{-4}^{0}$$

$$=74\left(\tfrac{2}{3}\right)(16)^{3/2}$$

$$=74\left(\tfrac{2}{3}\right)(64)=\frac{9472}{3}$$

$$=3157+\tfrac{1}{3}$$ lbs.

2. Suppose the average waiting time for a customer's call to be answered by a company representative (modeled by an exponential density function) is 10 minutes. Find the median waiting time. Find the probability that it takes more than 15 minutes for the call to be answered.

4 $\quad f(t)=\frac{1}{10}e^{-\frac{1}{10}t} \quad t\geq 0$

Find m so that $\int_0^m \frac{1}{10}e^{-\frac{1}{10}t}\,dt=\frac{1}{2}$ 2

$-e^{-\frac{1}{10}t}\Big|_0^m=\frac{1}{2}$ 2

8 $\quad -e^{-\frac{m}{10}}+1=\frac{1}{2}$ 2

$e^{-\frac{m}{10}}=\frac{1}{2}$

$-\frac{m}{10}=\ln\frac{1}{2}$

$m=-10\ln\frac{1}{2}=10\ln 2\approx 6.93$ min. 2

8 $\quad P(t\geq 15)=\int_{15}^{+\infty}\frac{1}{10}e^{-\frac{1}{10}t}\,dt=\lim_{r\to+\infty} -e^{-\frac{r}{10}}+e^{-\frac{15}{10}}=e^{-3/2}\approx .223$

4 2 2

$\frac{1}{2.25} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$

20 3. Find the length of the curve $y = x^{1.5}$, $0 \le x \le 5$.

$$L = \int_0^5 \sqrt{1+(f'(x))^2}\,dx \quad 4$$

$$= \int_0^5 \sqrt{1+(1.5x^{.5})^2}\,dx \quad 4$$

$$= \int_0^5 \sqrt{1+2.25x}\,dx \quad 4 \quad = \frac{1}{2.25}\frac{2}{3}(1+2.25x)^{3/2}\Big|_0^5 \quad 2 \quad 2 \quad 2$$

$$= \frac{2}{3}(12.25^{3/2}-1)\frac{1}{2.25} \quad 2 \quad = \frac{2}{3}(41.875)\frac{1}{2.25} = 12.41$$

20 4. Find the area of the surface obtained by rotating the curve $y = x^3$, $2 \le x \le 5$; about the *x*-axis. $S = \int_2^5 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx \quad 4$

$\frac{2}{3}\left(\left(\frac{5}{3}\right)^3 - 1\right)$

$= \frac{2}{3}\left(\frac{7^3}{8} - 8\right)$

$$S = \int_2^5 2\pi x^3 \sqrt{1+(3x^2)^2}\,dx \quad 4$$

$$= \int_2^5 2\pi x^3 \sqrt{1+9x^4}\,dx \quad 4$$

let $u = 1+9x^4$
$du = 36x^3\,dx$
$\frac{1}{36}du = x^3\,dx$

$$= \int_{145}^{5626} 2\pi\, u^{\frac{1}{2}}\, \frac{1}{36}\,du \quad 2$$

$$= \frac{\pi}{18}\frac{2}{3}u^{3/2}\Big|_{145}^{5626} \quad 2 \quad = \frac{\pi}{27}\left(5626^{3/2} - 145^{3/2}\right) = 48897.32$$

2

(20) 5. Find the centroid (center of mass) of the region bounded by the curves: $y = 4 - x$ and $y = x$, and $y = 0$.

$$4 \quad Mass = \int_0^2 \rho(4-x-x)\,dx = \int_0^2 \rho(4-2x)\,dx$$
$$= \rho(4x - x^2)\Big|_0^2 = \rho\, 4 = 4\rho$$

$$4\ M_y = \int_0^2 \rho\, x(4-x-x) = \int_0^2 \rho(4x-2x^2)\,dx$$
$$= \rho\left(2x^2 - \frac{2}{3}x^3\right)\Big|_0^2 = \rho\left(8 - \frac{2}{3}8\right) = \rho\frac{8}{3}$$

$$4\ M_x = \int_0^2 \frac{\rho}{2}\left((4-x)^2 - x^2\right)dx = \frac{\rho}{2}\int_0^2 (16-8x)\,dx$$
$$= \frac{\rho}{2}(16x - 4x^2)\Big|_0^2 = \frac{\rho}{2}(32-16) = \frac{\rho}{2}16 = 8\rho$$

(3) $\bar{x} = \frac{M_y}{Mass} = \frac{\rho\frac{8}{3}}{4\rho} = \frac{2}{3}$

(3) $\bar{y} = \frac{M_x}{Mass} = \frac{8\rho}{4\rho} = 2$

6. Let $f(x) = ke^{-6x}$ for $x \geq 4$ (for $x < 4$, $f(x) = 0$).

a. Find the value of k in order that f(x) is a probability density function.

$1 = \int_4^{+\infty} k e^{-6x}\, dx$ 3

$= \lim_{r \to +\infty} -\frac{1}{6} k e^{-6x} \Big|_4^r$ 3

10

$= \frac{1}{6} k e^{-24} \Rightarrow \boxed{k = 6e^{24}}$

3

$= \frac{6}{e^{-24}}$

10

b. Find the mean of the probability density function.

$\mu = \int_4^{+\infty} 6e^{24} x e^{-6x}\, dx =$

$\int \underbrace{x}_{u} \underbrace{e^{-6x} dx}_{dv} = -\frac{1}{6} x e^{-6x} - \int -\frac{1}{6} e^{-6x} dx = -\frac{1}{6} x e^{-6x} - \frac{1}{36} e^{-6x} + c$

6

$du = dx \quad v = -\frac{1}{6} e^{-6x}$

$\therefore = 6e^{24} \lim_{r \to +\infty} \left(-\frac{1}{6} x e^{-6x} - \frac{1}{36} e^{-6x}\right)\Big|_4^r$

$= 6e^{24}\left(\frac{4}{6} e^{-24} + \frac{1}{36} e^{-24}\right) = \left(\left(\frac{2}{3} + \frac{1}{36}\right) e^{-24}\right) 6e^{24}$

4

$= 4 + \frac{1}{6} =$

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MAT 162002 Test 2 Version 2 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book. Each problem is worth 20 points. Problem 6 is 20 points extra credit.

1. A cylindrical tank 10 feet in diameter is lying on its side and is half full of oil of density 76 lbs/ft^3 . What is the hydrostatic force on one end of the tank?

$x^2+y^2=5^2$

$x=\sqrt{25-y^2}$

$w(y)=2x$

$=2\sqrt{25-y^2}$

depth $=-y$

$\int_{-5}^{0} 76(-y)2\sqrt{25-y^2}\,dy = 76\left(\frac{2}{3}\right)(25-y^2)^{3/2}\Big|_{-5}^{0}$

$= 76\left(\frac{2}{3}\right)(25)^{3/2}$

$= 76\left(\frac{2}{3}\right)(125) = \frac{19000}{3}$

$= 6333\frac{1}{3}$ lbs.

2. Suppose the average waiting time for a customer's call to be answered by a company representative (modeled by an exponential density function) is 12 minutes. Find the median waiting time. Find the probability that it takes more than 15 minutes for the call to be answered.

4 $f(t)=\frac{1}{12}e^{-\frac{1}{12}t}$, $t\geq 0$

Find M so that $\int_0^M \frac{1}{12}e^{-\frac{1}{12}t} = \frac{1}{2}$ 2

$-e^{-\frac{1}{12}t}\Big|_0^M = \frac{1}{2}$ 2

$-e^{-\frac{1}{12}M}+1=\frac{1}{2}$ 2

8

$e^{-\frac{M}{12}}=\frac{1}{2}$

$-\frac{M}{12}=\ln\frac{1}{2}$ 2

$M=-12\ln\frac{1}{2}=12\ln 2 \approx 8.32$ min.

$P(t\geq 15)=\int_{15}^{+\infty}\frac{1}{12}e^{-\frac{1}{12}t}\,dt$ 2

$=\lim_{r\to+\infty} -e^{-\frac{r}{12}}+e^{-\frac{15}{12}} = e^{-\frac{15}{12}}=e^{-\frac{5}{4}}$

8 2

$\approx .2857$

3. Find the length of the curve $y = 4x^{1.5}, 0 \le x \le 5$.

$$L = \int_0^5 \sqrt{1+(f'(x))^2}\,dx \quad 4$$

$$= \int_0^5 \sqrt{1+(6x^{\frac{1}{2}})^2}\,dx \quad 4$$

$$= \int_0^5 \sqrt{1+36x}\,dx \quad 4 \quad = \frac{2}{3}\frac{1}{36}(1+36x)^{3/2}\Big|_0^5 \quad 20$$

$$= \frac{1}{54}(181^{3/2}-1) \quad 2 \quad = 45.076$$

4. Find the area of the surface obtained by rotating the curve $y = 2x^3$, $2 \le x \le 5$, about the x-axis.

$$S = \int_2^5 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx \quad 4$$

$$S = \int_2^5 2\pi\, 2x^3\sqrt{1+(6x^2)^2}\,dx \quad 4$$

$$= \int_2^5 4\pi\, x^3\sqrt{1+36x^4}\,dx \quad 4 \qquad 2$$

let $u = 1+36x^4$

$du = 4(36)x^3\,dx$

$\frac{1}{36}du = 4x^3\,dx$

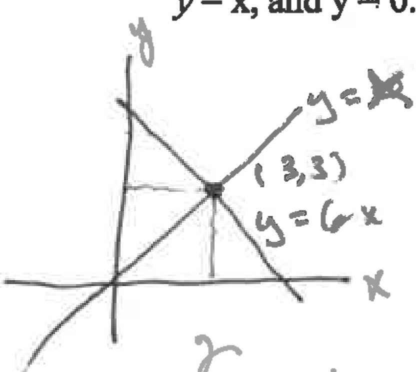
$$= \int_{577}^{22501} \frac{\pi}{36} u^{\frac{1}{2}}\,du \quad 2$$

$$= \frac{\pi}{36}\frac{2}{3}u^{3/2} \quad 2$$

$$= \frac{1}{54}\pi(22501^{3/2} - 577^{3/2})$$

$= 219{,}556.29$

5. Find the centroid (center of mass) of the region bounded by the curves: $y = 6 - x$ and $y = x$, and ~~$y = 0$~~ $x=0$.



2

$$4 \quad \text{Mass} = \int_0^3 \rho(6-x-x)\,dx = \int_0^3 \rho(6-2x)\,dx$$

$$= \rho(6x-x^2)\Big|_0^3 = \rho(18-9) = 9\rho$$

$$4 \quad M_y = \int_0^3 \rho x(6-2x)\,dx = \int_0^3 \rho(6x-2x^2)\,dx$$

$$= \rho(3x^2 - \tfrac{2}{3}x^3)\Big|_0^3 = \rho(27-18) = 9\rho$$

$$4 \quad M_x = \int_0^3 \frac{\rho}{2}((6-x)^2-x^2)\,dx = \int_0^3 \frac{\rho}{2}(36-12x)\,dx$$

$$= \frac{\rho}{2}(36x-6x^2)\Big|_0^3 = \frac{\rho}{2}(108-54) = 27\rho$$

$$3\ \bar{x} = \frac{M_y}{\text{Mass}} = \frac{9\rho}{9\rho} = 1 \qquad 3\ \bar{y} = \frac{M_x}{\text{Mass}} = \frac{27\rho}{9\rho} = 3$$

6. Let $f(x) = ke^{-3x}$ for $x \geq 5$ (for $x < 5$, $f(x) = 0$).

a. Find the value of k in order that f(x) is a probability density function.

$$1 = \int_5^{+\infty} ke^{-3x}\,dx = \lim_{r \to +\infty} -\frac{1}{3}ke^{-3x}\Big|_5^r = \frac{1}{3}ke^{-15}$$

3

$$\therefore k = 3e^{15} = \frac{3}{e^{-15}}$$

10

b. Find the mean of the probability density function.

10

$$\mu = \int_5^{+\infty} 3e^{15}x\,e^{-3x}\,dx$$

$$\int \underbrace{x}_{u}\underbrace{e^{-3x}dx}_{dv} = \left(-\frac{1}{3}xe^{-3x} - \int\left(-\frac{1}{3}e^{-3x}\right)dx\right)$$

$$= \left(-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}\right)3e^{15}$$

$du = dx \quad v = -\frac{1}{3}e^{-3x}$

$$\therefore \mu = 3e^{15}\lim_{r \to +\infty}\left(-\frac{1}{3}re^{-3r} - \frac{1}{9}e^{-3r} + \frac{1}{3}5e^{-15} + \frac{1}{9}e^{-15}\right)$$

$$= 3e^{15}\left(\frac{5}{3}e^{-15} + \frac{1}{9}e^{-15}\right)$$

$$= 5 + \frac{1}{3}$$

$$\mu = 5 + \frac{1}{3}$$