Fall, 2017 Test 1 Name: Key

MAT 162001 + 002 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book. When you turn in your test, staple your notes to this sheet.

For 1-4, calculate the following:

10 1. $\int x\sec^2(x)\,dx = x\tan x - \int \tan x\,dx = x\tan x - (\ln|\sec x|) + C$

$= \boxed{x\tan x + \ln|\cos x| + C}$

$u = x \quad dv = \sec^2 x\,dx$

$du = dx \quad v = \tan x$

10 2. $\int \sin^2(x)\cos^3(x)\,dx = \int \sin^2 x\,(1-\sin^2 x)\cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int u^2(1-u^2)\,du$

$= \int (u^2 - u^4)\,du = \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$

10 3. $\int x^3\sqrt{16+x^2}\,dx = \int 4^3\tan^3\theta \; 4\sec\theta \; 4\sec^2\theta\,d\theta$

$x = 4\tan\theta$

$\sqrt{16+x^2} = 4\sec\theta$

$dx = 4\sec^2\theta\,d\theta$

$= 4^5\int \tan^2\theta\sec^2\theta\,\tan\theta\sec\theta\,d\theta$

$= 4^5\int (\sec^2\theta - 1)\sec^2\theta\,\tan\theta\sec\theta\,d\theta$

$= 4^5\int (w^2-1)w^2\,dw$

$= 4^5\int w^4 - w^2\,dw$

$w = \sec\theta$

$dw = \sec\theta\tan\theta\,d\theta$

$= 4^5\left(\frac{1}{5}w^5 - \frac{1}{3}w^3\right) + C$

$= 4^5\left(\frac{1}{5}\frac{(x^2+16)^{5/2}}{4^5} - \frac{1}{3}\frac{(x^2+16)^{3/2}}{4^3}\right) + C$

$\sec\theta = \frac{\sqrt{x^2+16}}{4}$

$= \frac{1}{5}(x^2+16)^{5/2} - \frac{1}{3}4^2(x^2+16)^{3/2} + C$

2 2

4. $\int \frac{x+2}{x^2+8x+17}dx$ $= \int \frac{x+2}{(x+4)^2+1}dx = \int \frac{x+4-2}{(x+4)^2+1}dx$

10

$= \frac{1}{2}\int \frac{2(x+4)}{(x+4)^2+1}dx - 2\int \frac{1}{(x+4)^2+1}dx$ 2

$= \frac{1}{2}\ln(x^2+8x+17) - 2\arctan(x+4) + C$

2 2

5. Estimate $\int_4^6 e^{1+\sin(x)}dx$ using the Trapezoidal Rule with n = 6. Write the sum; you do not have to evaluate the sum.

8

Let $f(x) = e^{1+\sin(x)}$

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$n=6$

4 ⊢——⊣ 6

$\Delta x = \frac{6-4}{6} = \frac{1}{3}$

$\frac{\frac{1}{3}}{2}\left[f(4) + 2f(4+\tfrac{1}{3}) + 2f(4+\tfrac{2}{3}) + 2f(5) + 2f(5+\tfrac{1}{3}) + 2f(5+\tfrac{2}{3}) + f(6)\right]$

6. Calculate the following; it the integral does not converge, state "does not converge."

8 a. $\int_6^{+\infty} \frac{1}{x^2+2x-15}dx$ $= \lim_{t\to+\infty} \int_6^t \frac{1}{8}\left[\frac{1}{x-3} - \frac{1}{x+5}\right]dx$ 2

$\frac{1}{x^2+2x-15} = \frac{A}{x+5} + \frac{B}{x-3}$

$1 = A(x-3) + B(x+5)$

$x=-5 \quad 1 = A(-8) \Rightarrow A = -\frac{1}{8}$

$x=3 \quad 1 = B(8) \Rightarrow B = \frac{1}{8}$

$= \lim_{t\to+\infty} \frac{1}{8}\left[\ln(t-3) - \ln(t+5) - \ln 3 + \ln 11\right]$ 1

$= \lim_{t\to+\infty} \frac{1}{8}\left[\ln\left|\frac{t-3}{t+5}\right| + \ln\frac{11}{3}\right]$

$= \boxed{\frac{1}{8}\ln\frac{11}{3}}$ 2

b. $\int_3^5 \frac{4}{\sqrt{x-3}}dx$

8

$\lim_{t\to3^+} \int_t^5 \frac{4}{\sqrt{x-3}}dx = \lim_{t\to3^+} 2(4)\sqrt{x-3}\Big|_t^5$ 2 2

2

$= 2(4)\sqrt{2}$ 2

$= 8\sqrt{2}$

7. Tell why the following converge or diverge:

8

a. $\int_1^{+\infty} \frac{1+\cos^2(x)}{x^2+2}\,dx$

$\frac{1+\cos^2 x}{x^2+2} \le \frac{2}{x^2}$ 3

$\int_1^{+\infty} \frac{2}{x^2}\,dx$ converges $(p>1)$ 3

$\therefore \int_1^{+\infty} \frac{1+\cos^2 x}{x^2+2}\,dx$ converges 2

8

b. $\int_1^{+\infty} \frac{1+\cos^2(x)}{x+2}\,dx$

$\frac{1}{3}\frac{1}{x} \le \frac{1}{x+2x} \le \frac{1+\cos^2 x}{x+2}$ 3

$\int_1^{+\infty} \frac{1}{3}\frac{1}{x}\,dx$ diverges $(p=1)$ 3

$\therefore \int_1^{+\infty} \frac{1+\cos^2 x}{x+2}$ diverges 2

8. Calculate $\int \frac{x+2}{x^2+8x+15}\,dx$

10

$\frac{x+2}{(x+5)(x+3)}$ 2 $= \frac{A}{x+5} + \frac{B}{x+3}$ 2

$x+2 = A(x+3) + B(x+5)$

$x=-5 \quad -3 = A(-2) \Rightarrow A = \frac{3}{2}$

$x=-3 \quad -1 = B(2) \Rightarrow B = -\frac{1}{2}$ 2

$= \int \frac{3}{2}\frac{1}{x+5} - \frac{1}{2}\frac{1}{x+3}\,dx$ 1 1

$= \frac{3}{2}\ln|x+5| - \frac{1}{2}\ln|x+3| + C$

1 1 1

9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the integral): $\int \frac{2x+3}{(x^2-1)^3(x^2+2x+5)^2}\,dx$

10

$\frac{2x+3}{(x-1)^3(x+1)^3\left((x+1)^2+4\right)^2}$

1

$= \frac{A_1}{x-1} + \frac{A_2}{(x-1)^2} + \frac{A_3}{(x-1)^3} + \frac{B_1}{x+1} + \frac{B_2}{(x+1)^2} + \frac{B_3}{(x+1)^3}$

$+ \frac{C_1x+D_1}{x^2+2x+5} + \frac{C_2x+D_2}{(x^2+2x+5)^2}$

1 1