Fall, 2014 Name: Key

MAT 162002 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator and notes recorded on one side of a single 8.5 by 11 inch paper. Otherwise the test is closed book. When you turn in your test, staple your notes to this sheet.

For 1-4, calculate the following:

1. $\int x^2 \sin(5x)\ dx = -\frac{1}{5}x^2\cos(5x) - \int -\frac{1}{5}\cos(5x)\, 2x\,dx$

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$u = x^2 \quad dv = \sin(5x)\,dx$

$du = 2x\,dx \quad v = -\frac{1}{5}\cos(5x)$

$= -\frac{1}{5}x^2\cos(5x) + \frac{2}{5}\int x\cos(5x)\,dx$

$= -\frac{1}{5}x^2\cos(5x) + \frac{2}{5}\left(x\frac{\sin(5x)}{5} - \int \frac{\sin(5x)}{5}\,dx\right)$

$= -\frac{1}{5}x^2\cos(5x) + \frac{2}{25}x\sin(5x) + \frac{2}{5^3}\cos(5x) + C$

2. $\int \tan^6(x)\sec^4(x)\,dx = \int \tan^6 x\,(1+\tan^2 x)\sec^2 x\,dx$ 2

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$w = \tan x$

$dw = \sec^2 x\,dx$

$= \int w^6(1+w^2)\,dw$ 2

$= \frac{1}{7}w^7 + \frac{1}{9}w^9 + C$ 2

$= \boxed{\frac{1}{7}\tan^7 x + \frac{1}{9}\tan^9 x + C}$ 2

3. $\int e^x\sqrt{100-e^{2x}}\,dx = \int \sqrt{100-w^2}\,dw = \int 10\cos\theta\ 10\cos\theta\,d\theta$

$w = e^x$

$dw = e^x\,dx$

$w = 10\sin\theta$

$dw = 10\cos\theta\,d\theta$

$\sqrt{100-w^2} = 10\cos\theta$

$= 100\int \frac{1+\cos 2\theta}{2}\,d\theta$ $\frac{1}{2}$

$= 50\left(\theta + \frac{\sin 2\theta}{2}\right) + C$

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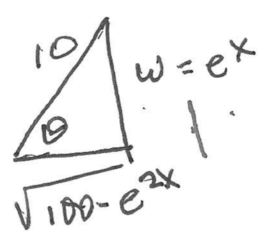
$= 50\left(\arcsin\frac{e^x}{10} + \frac{e^x\sqrt{100-e^{2x}}}{100}\right) + C$



$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

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2

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4. $\int \frac{2x}{x^2+2x-35}dx = \int \frac{2x}{(x+7)(x-5)}dx = \int \left(\frac{7}{6}\frac{1}{x+7} + \frac{5}{6}\frac{1}{x-5}\right)dx$

2 $\frac{2x}{(x+7)(x-5)} = \frac{A}{x+7} + \frac{B}{x-5}$ 2

$= \frac{7}{6}\ln|x+7| + \frac{5}{6}\ln|x-5|$

$2x = A(x-5) + B(x+7)$ 2

$10 = B(12) \Rightarrow B = \frac{10}{12}$

$-14 = A(-12) \Rightarrow A = \frac{14}{12}$

2 $+C$

5. Estimate $\int_1^3 e^{\sin(x)}dx$ using the Simpson's Rule with n = 8. Write the sum; you do not have to evaluate the sum.

$\Delta x = \frac{3-1}{8} = \frac{1}{4}$ 1 $f(x) = e^{\sin x}$ 1

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$\frac{\frac{1}{4}}{3}$ 1 $\Big(f(1) + 4f(\frac{5}{4}) + 2f(\frac{6}{4}) + 4f(\frac{7}{4}) + 2f(\frac{8}{4}) + 4f(\frac{9}{4}) + 2f(\frac{10}{4}) + 4f(\frac{11}{4}) + f(3) \Big)$ 1

6. Calculate the following; it the integral does not converge, state "does not converge."

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a. $\int_1^{+\infty} x^3/(1+x^4)dx = \lim_{b\to+\infty} \frac{1}{4}\ln(1+x^4)\Big|_1^b = \lim_{b\to+\infty} \frac{1}{4}\ln(1+b^4) - \ln 2$ 2

2 $= +\infty$ 2

does not converge 2

8

b. $\int_3^4 \frac{3}{\sqrt{4-x}}dx = \lim_{b\to 4^-} \int_3^b \frac{3}{\sqrt{4-x}}dx = \lim_{b\to4^-} -6\sqrt{4-x}\Big|_3^b$ 2

2

$= 6$ 4

7. Tell why the following converge or diverge:

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a. $\int_1^{+\infty} \frac{x^3+1}{(x^2+24)^2}dx$ $\quad \frac{x^3}{x^4} = \frac{1}{x}$

$\frac{1}{25}\frac{1}{x}$

$\frac{x^3}{(x^2+24x^2)^2} \le \frac{x^3+1}{(x^2+24)^2}$ 2

$\int_1^{+\infty} \frac{1}{25}\frac{1}{x}dx$ diverges 2 $\Rightarrow \int_1^{+\infty} \frac{x^3+1}{(x^2+24)^2}dx$ 2 diverges 2

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b. $\int_1^{+\infty} \frac{x^3+1}{(x+24)^6}dx$

2 $\frac{x^3+1}{(x+24)^6} \le \frac{x^3+x^3}{x^6} = \frac{2}{x^3}$

2 $\int_1^{+\infty} \frac{2}{x^3}dx$ ~~diverges~~ converges 2

$\therefore \int_1^{+\infty} \frac{x^3+1}{(x+1)^{4}}$ 2 converges

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10

8. Calculate $\int \frac{2x}{x^2+2x+37}dx$

$$\int \frac{2x}{x^2+2x+1+6^2}dx = \int \frac{2x}{(x+1)^2+6^2}dx \quad 2$$

$$= \int \frac{2(x+1)}{(x+1)^2+6^2}dx \quad 2 \quad -2\int \frac{1}{(x+1)^2+4^2}dx \quad 2$$

$$= \ln\left((x+1)^2+6^2\right) - \frac{2}{6}\arctan\frac{x+1}{6} + C$$

2 2

9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the integral): $\int \frac{4x+5}{(x^2+12x+40)^3(x^2+3x-10)^2}dx$

$(x+6)^2+2^2 \quad (x+5)(x-2)$

$$2\ \frac{4x+5}{((x+6)^2+2^2)^3\,(x+5)^2(x-2)^2} \quad 2$$

$$\frac{A_1x+B_1}{(x+6)^2+2^2} + \frac{A_2x+B_2}{((x+6)^2+2^2)^2} + \frac{A_3x+B_3}{((x+6)^2+6^2)^3} \quad 2$$

$$+ \frac{C_1}{x+5} + \frac{C_2}{(x+5)^2} \quad 2$$

$$+ \frac{D_1}{x-2} + \frac{D_2}{(x-2)^2} \quad 2$$

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