Fall, 2014 Name:____________________________

MAT 162001 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator and notes recorded on one side of a single 8.5 by 11 inch paper. Otherwise the test is closed book. When you turn in your test, staple your notes to this sheet.

For 1-4, calculate the following:

1. $\int x^2\cos(3x)\,dx$ $= \frac{x^2\sin(3x)}{3} - \frac{2}{3}\int x\sin(3x)\,dx$

$u = x^2$, $du = 2x\,dx$ $\quad$ $dv = \cos(3x)\,dx$, $v = \frac{\sin(3x)}{3}$

$= \frac{1}{3}x^2\sin(3x) - \frac{2}{3}\left[\frac{-x\cos(3x)}{3} + \int\frac{\cos(3x)}{3}dx\right]$

$= \frac{1}{3}x^2\sin(3x) - \frac{2}{3}\left[-\frac{x\cos(3x)}{3} + \frac{\sin(3x)}{9}\right] + C$

check: $x^2\cos(3x) + \frac{2}{3}x\cancel{\sin(3x)} - \frac{2}{3}\left(-\frac{1}{3}\cancel{\cos 3x} + x\cancel{\sin(3x)} + \cancel{\frac{\cos(3x)}{3}}\right)$

2. $\int \tan^3(x)\sec^3(x)\,dx$ $= \int \tan^2 x\sec^2 x\tan x\sec x\,dx$ 2

$= \int(\sec^2 x - 1)\sec^2 x\tan x\sec x\,dx$ 2

$w = \sec x$

$dw = \sec x\tan x\,dx$

$= \int(w^2-1)w^2\,dw = \int(w^4 - w^2)\,dw$ 2

$= \boxed{\frac{1}{5}w^5 - \frac{1}{3}w^3 + C} = \boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$

2

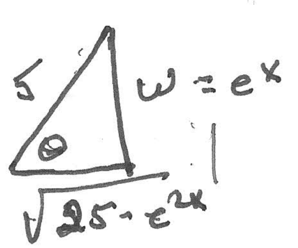
3. $\int e^x\sqrt{25-e^{2x}}\,dx$ $= \int\sqrt{25-w^2}\,dw = \int 5\cos\theta\, 5\cos\theta\,d\theta$

$w = e^x$

$dw = e^x\,dx$

$w = 5\sin\theta$

$= \int 25\left(\frac{1+\cos 2\theta}{2}\right)d\theta$ $\frac{1}{2}$



$= \frac{25}{2}\left(\theta + \frac{\sin(2\theta)}{2}\right) + C$ $\frac{1}{2}$

$= \frac{25}{2}(\theta + \sin\theta\cos\theta) + C$

$= \frac{25}{2}\left(\arcsin\frac{e^x}{5} + \frac{e^x\sqrt{25-e^{2x}}}{25}\right) + C$

$\frac{1}{2}$ $\frac{1}{2}$

$\frac{1}{30}$

4. $\int \frac{2x}{x^2-2x-35}dx$ $= \int \frac{7}{6}\frac{1}{x-7} + \frac{5}{6}\frac{1}{x+5}\,dx$ 2

$\frac{2x}{(x-7)(x+5)} = \frac{A}{x-7} + \frac{B}{x+5}$ 2 2

$= \frac{7}{6}\ln|x-7| + \frac{5}{6}\ln|x+5| + C$ 2

10

$2x = A(x+5) + B(x-7)$

$x=7$ $14 = A(12) \Rightarrow A = \frac{14}{12} = \frac{7}{6}$ 2

$x=-5$ $-10 = B(-12) \Rightarrow B = \frac{10}{12} = \frac{5}{6}$

5. Estimate $\int_1^3 e^{\sin(x)}dx$ using the Simpson's Rule with n = 8. Write the sum; you do not have to evaluate the sum.

8

$x_i = 1 + i\frac{1}{4}$ $0 \le i \le 8$ 1

1 ——— 3 1

$\frac{\Delta x}{3} = \frac{\frac{1}{4}}{3} = \frac{1}{12}$ 1

$\Delta x = \frac{b-a}{n} = \frac{3-1}{8} = \frac{2}{8} = \frac{1}{4}$ 1

$\frac{1}{12}\left(f(1) + 4f(\tfrac{5}{4}) + 2f(\tfrac{6}{4}) + 4f(\tfrac{7}{4}) + 2f(\tfrac{8}{4}) + 4f(\tfrac{9}{4}) + 2f(\tfrac{10}{4}) + 4f(\tfrac{11}{4}) + f(3)\right)$ 1 1 1 1 1

6. Calculate the following; it the integral does not converge, state "does not converge." 1

a. $\int_1^{+\infty} x^3/(1+x^4)dx$ $= \lim_{b\to+\infty} \int_1^b \frac{x^3}{1+x^4}dx$ 2

8

"does not converge" 2

$= \lim_{b\to+\infty} \frac{1}{4}\ln(1+x^4)\Big|_1^b$ 2

$= \lim_{b\to+\infty} \frac{1}{4}\ln(1+b^4) - \frac{1}{4}\ln 2 = +\infty$

2

b. $\int_3^4 \frac{3}{\sqrt{4-x}}dx$ $= \lim_{b\to4^-} \int_3^b \frac{3}{\sqrt{4-x}}dx$ 2

8

$= \lim_{b\to4^-} -6(4-x)^{\frac{1}{2}}\Big|_3^b$ 2

$= 0 + 6(1^{\frac{1}{2}}) = 6$ 4

7. Tell why the following converge or diverge:

8

a. $\int_1^{+\infty} \frac{x^3+1}{(x^2+24)^2}dx$ $\frac{x^3}{x^4} \approx \frac{1}{x}$

$\frac{1}{25x} \le \frac{x^3}{(x^2+24x^2)^2} \le \frac{x^3+1}{(x^2+24)^2}$ 4

$\int_1^{+\infty} \frac{1}{25x}dx$ diverges $\Rightarrow \int_1^{+\infty} \frac{x^3+1}{(x^2+24)^2}dx$ diverges

since $p=1$

2 2

8

b. $\int_1^{+\infty} \frac{x^3+1}{(x+24)^6}dx$ $\frac{x^3}{x^6} = \frac{1}{x^3}$

$\frac{x^3+1}{(x+24)^6} \le \frac{2x^3}{x^6} \le \frac{2}{x^3}$ 4

$\int_1^{+\infty} \frac{2}{x^3}dx$ converges since $p \geq 3$ $\Rightarrow \int_1^{+\infty} \frac{x^3+1}{(x+24)^6}dx$ converges

2 2

/50

8. Calculate $\int \frac{2x}{x^2-2x+50}\,dx$

3

10

$$\frac{2x}{x^2-2x+1+7^2} = \frac{2x}{(x-1)^2+7^2} = \frac{2(x-1)}{(x-1)^2+7^2} + \frac{2}{(x-1)^2+7^2}$$

$$\int \frac{2(x-1)}{(x-1)^2+7^2}\,dx + 2\int \frac{1}{(x-1)^2+7^2}\,dx$$

2

$$= \boxed{\ln(x^2-2x+50) + \frac{2}{7}\arctan\left(\frac{x-1}{7}\right)+C}$$

2 1

2

9. Write the form of the partial fraction decomposition that you would use to calculate the following integral (you do not have to solve for the constants nor evaluate the integral): $\int \frac{4x+5}{(x^2+12x+40)^3(x^2+3x-10)^2}\,dx$

10

$$\frac{4x+5}{((x+6)^2+2^2)^3(x+5)^2(x-2)^2}$$ 2

2

$$= \frac{Ax_1+B_1}{x^2+12x+40} + \frac{Ax_2+B_2}{(x^2+12x+40)^2} + \frac{Ax_3+B_3}{(x^2+12x+40)^3}$$ 2

$$+ \frac{C_1}{x+5} + \frac{C_2}{(x+5)^2} + \frac{D_1}{x-2} + \frac{D_2}{(x-2)^2}$$

2

2

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