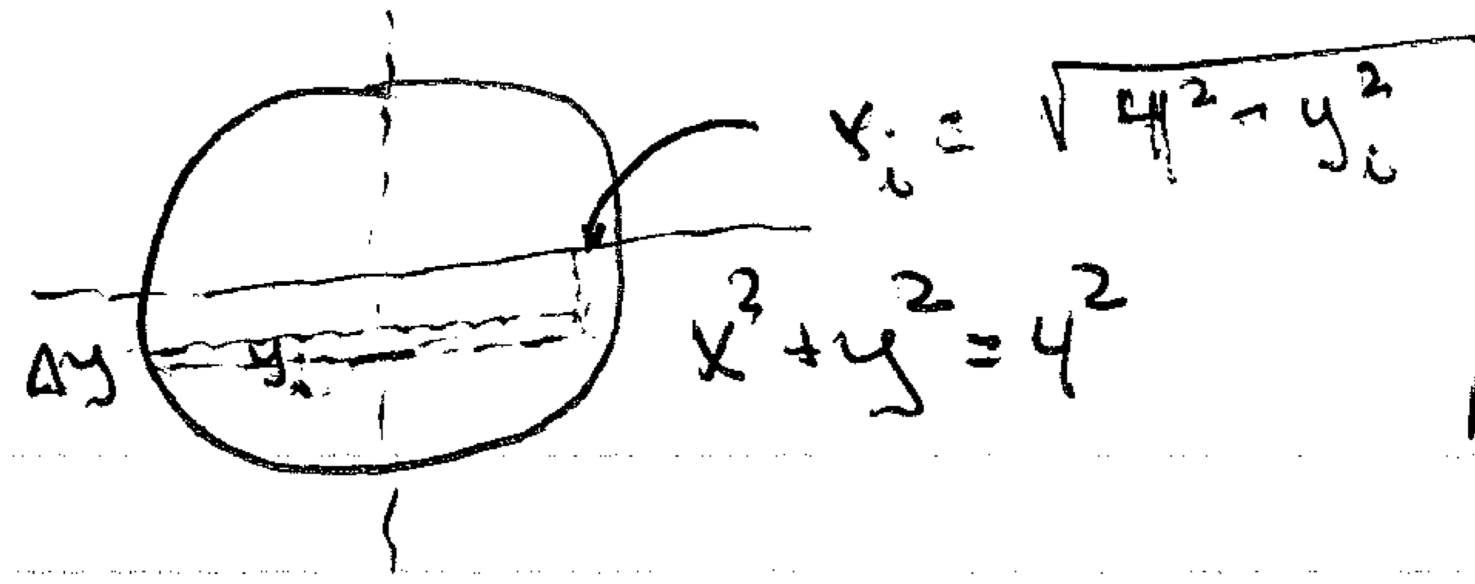
① 

Area $= 2\sqrt{4^2 - y_i^2}\;\Delta y$

depth $= -y_i$

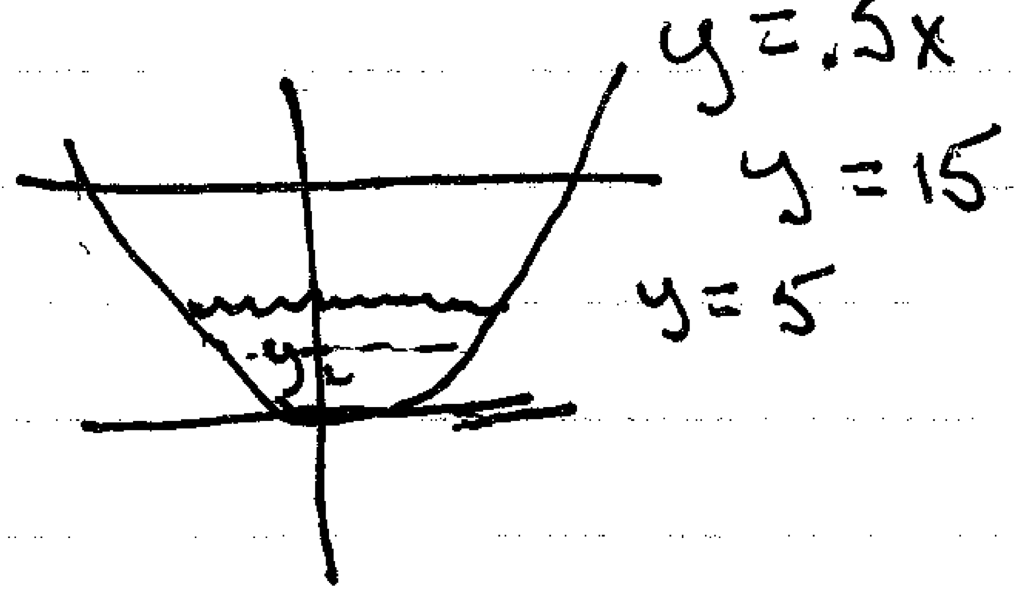
$$\lim_{\Delta y \to 0} \sum_{i=1}^{n} 75(-y_i)\,2\sqrt{4^2 - y_i^2}\,\Delta y = \int_{-4}^{0} -75y\,2\sqrt{4^2 - y^2}\,dy$$

$$= 75\left(\tfrac{2}{3}\right)(4^2 - y^2)^{3/2}\Big|_{-4}^{0}$$

$$= 50(4^2 - y^2)^{3/2}\Big|_{-4}^{0}$$

$$= 50\left[(4^2)^{3/2} - (4^2 - 4^2)^{3/2}\right]$$

$$= 50\left[4^3\right] = 3200 \text{ lb.}$$

② 

$y_i = .5x_i^2$

$2y_i = x_i^2$

$x_i = \sqrt{2}\;y_i^{\frac{1}{2}}$

Area $= 2\sqrt{2}\,y^{\frac{1}{2}}\,\Delta y$

depth $= 5 - y_i$

$$\lim_{\Delta y \to 0} \sum_{i=1}^{n} 42(5 - y_i)\,2\sqrt{2}\,y_i^{\frac{1}{2}}\,\Delta y = \int_{0}^{5} 84\sqrt{2}\,(5 - y)\,y^{\frac{1}{2}}\,dy$$

$$= 84\sqrt{2}\int_0^5 \left(5y^{\frac{1}{2}} - y^{3/2}\right)dy$$

$$= 84\sqrt{2}\left(5\,\tfrac{2}{3}y^{3/2} - \tfrac{2}{5}y^{5/2}\right)\Big|_0^5$$

$$= 84\sqrt{2}\left(\tfrac{10}{3}5^{3/2} - \tfrac{2}{5}5^{5/2}\right)$$

$$= 84\sqrt{2}\left(\tfrac{2}{3}5^{5/2} - \tfrac{2}{5}5^{5/2}\right)$$

$$= 84\sqrt{2}\left(\tfrac{4}{15}\right)5^{5/2} = 84\sqrt{2}\,\tfrac{4}{3}\,5^{3/2}$$

$$= 108\sqrt{2}\;5^{3/2}$$

$$= 540\sqrt{10} \text{ lbs.}$$

(3) $f(x) = \frac{1}{6}(x^2+4)^{1.5} \quad 0 \le x \le 2$

$$f'(x) = \frac{1}{6}(1.5)(x^2+4)^{.5}\,2x = \frac{1}{2}(x^2+4)^{.5}\,x$$

$$1+(f'(x))^2 = 1+\frac{1}{4}x^2(x^2+4) = \frac{1}{4}x^4+x^2+1 = \left(\frac{1}{2}x^2+1\right)^2$$

$$\sqrt{1+(f'(x))^2} = \frac{1}{2}x^2+1$$

$$\int_0^2 \sqrt{1+(f'(x))^2}\,dx = \int_0^2 \left(\frac{1}{2}x^2+1\right)dx = \left(\frac{1}{6}x^3+x\right)\Big|_0^2$$

$$= \frac{1}{6}(8)+2 = \boxed{3\frac{1}{3}}$$

④ $f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x \qquad 1 \le x \le 2$

$$f'(x) = \frac{1}{2}x - \frac{1}{2}\frac{1}{x} = \frac{1}{2}\left(x - \frac{1}{x}\right)$$

$$\sqrt{1+(f'(x))^2} = \sqrt{1 + \frac{1}{4}\left(x^2 - 2 + \frac{1}{x^2}\right)} = \sqrt{\frac{1}{4}\left(x^2 + 2 + \frac{1}{x^2}\right)} = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

$$\int_1^2 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx = \int_1^2 2\pi\left(\frac{1}{4}x^2 - \frac{1}{2}\ln x\right)\left(\frac{1}{2}\right)\left(x + \frac{1}{x}\right)dx$$

$$= \int_1^2 \frac{\pi}{4}\left(x^2 - 2\ln x\right)\left(x + \frac{1}{x}\right)dx = \int_1^2 \frac{\pi}{4}\left(x^3 + x - 2\frac{1}{x}\ln x - 2\underline{x\ln x}\right)dx$$

Aside $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2\frac{1}{x}\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{2}x^2$

$$\therefore \int_1^2 \frac{\pi}{4}\left(x^3 + x - 2\frac{1}{x}\ln x - 2x\ln x\right)dx =$$

$$= \frac{\pi}{4}\left(\frac{1}{4}x^4 + \frac{1}{2}x^2 - (\ln x)^2 - x^2\ln x + \frac{1}{2}x^2\right)\Big|_1^2 =$$

$$= \frac{\pi}{4}\left(\frac{1}{4}x^4 + x^2 - (\ln x)^2 - x^2\ln x\right)\Big|_1^2 =$$

$$= \frac{\pi}{4}\left(\frac{1}{4}2^4 + 2^2 - (\ln 2)^2 - 4\ln 2 - \frac{1}{4} - 1\right)$$

$$= \frac{\pi}{4}\left(4 + 4 - (\ln 2)^2 - 4\ln 2 - \frac{1}{4} - 1\right)$$

$$= \boxed{\frac{\pi}{4}\left(6.75 - (\ln 2)^2 - 4\ln 2\right)}$$

or $2\pi\left(\frac{27}{32} - \frac{1}{2}\ln(2) - \frac{1}{8}(\ln 2)^2\right) \approx 3.219634$

⑤ $\mu = 10$ $\quad f(x) = \begin{cases} \frac{1}{10} e^{-\frac{1}{10}x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

Find M so that $\int_0^M \frac{1}{10} e^{-\frac{1}{10}x}\,dx = .5$

$$-e^{-\frac{1}{10}x}\Big|_0^M = .5$$

$$1 - e^{-\frac{M}{10}} = .5$$

$$.5 = e^{-\frac{M}{10}}$$

$$\ln .5 = -\frac{M}{10}$$

$$\boxed{M = -10 \ln .5 = 6.93 \text{ minutes}}$$

$$P(X > 15) = \int_{15}^{+\infty} \frac{1}{10} e^{-\frac{1}{10}x}\,dx$$

$$= 1 - \int_0^{15} \frac{1}{10} e^{-\frac{1}{10}x}\,dx$$

$$1 - \left(-e^{-\frac{1}{10}x}\right)\Big|_0^{15} = 1 - \left(1 - e^{-\frac{15}{10}}\right)$$

$$= e^{-\frac{3}{2}} = \boxed{.223}$$

⑥ $1 = \int_3^{+\infty} (kx^{-3} + x^{-5})\,dx = \lim_{b \to +\infty} \int_3^b (kx^{-3} + x^{-5})\,dx$

$$= \lim_{b \to +\infty} \left(-\tfrac{1}{2} k x^{-2} - \tfrac{1}{4} x^{-4}\right)\Big|_3^b = \frac{k}{2}\frac{1}{9} + \frac{1}{4}\frac{1}{81}$$

$$k = 18\left(1 - \tfrac{1}{4}\tfrac{1}{81}\right) = 18 - \tfrac{1}{2}\tfrac{1}{9} = 17\tfrac{17}{18} = \boxed{\frac{323}{18}} = 17.94$$

$$\mu = \int_3^{+\infty} (kx^{-2} + x^{-4})\,dx = \lim_{b \to +\infty} \int_3^b (kx^{-2} + x^{-4})\,dx$$

$$= \lim_{b \to +\infty} \left(-kx^{-1} - \tfrac{1}{3} x^{-3}\right)\Big|_3^b$$

$$= k\frac{1}{3} + \frac{1}{81} = \frac{323}{3(18)} + \frac{1}{81} = 5.9938$$

$$= \boxed{\frac{971}{162}}$$