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MAT 162 Test 4 Name Key

Fall 2014

For full credit, show all work.

1. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(n+1)(n^2-1)}{4n^3-2n+1}$ ~~comparison test~~

diverges since $\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \frac{(n+1)(n^2-1)}{4n^3-2n+1} = 4 \neq 0$.

12

6 answer

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{6n}$ converges by Alt. Series tests

but $\sum_{n=1}^{\infty} \left|\frac{-1}{6n}\right| = \sum_{n=1}^{\infty} \frac{1}{6n}$ diverges

12 $\therefore$ converges conditionally

(c) $\sum_{n=1}^{\infty} \frac{(-4)^{n+2}}{3^{2n+1}}$

$\sum_{n=1}^{\infty} \left|\frac{(-4)^{n+2}}{3^{2n+1}}\right| = \frac{4^2}{3} \sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^n$ geometric series $|r| = \frac{4}{9} < 1$

12 $\therefore$ converges absolutely

2. Find the radius and interval of convergence for $f(x) = \sum_{n=1}^{\infty}(x-5)^{2n}(9+4/n)^{-n}$.

13

$\lim_{n\to+\infty} |a_n|^{\frac{1}{n}} = \frac{|x-5|^2}{9} < 1$

$|x-5| < 3$

$R = 3$

$-3 < x-5 < 3$

$2 < x < 8$

$(2, 8)$

5

$x=8 \quad \sum_{n=1}^{\infty} \frac{3^{2n}}{\left|9+\frac{4}{n}\right|^n} = \sum_{n=1}^{\infty} \left(\frac{9}{9+\frac{4}{n}}\right)^n$

1

$\lim_{n\to+\infty} \frac{1}{\left(1+\frac{4}{9n}\right)^n} = \frac{1}{e^{n\ln\left(1+\frac{4}{9n}\right)}} \neq 0$

$x=3 \quad \sum \frac{(-3)^{2n}}{\left(9+\frac{4}{n}\right)^n}$

1 $\lim a_n \neq 0$

3. Use a power series to estimate $\int_0^{0.1} xe^{-x^3}\,dx$ with an error less than 10^{-6}.

13

$\int_0^{.1} \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{3n+1}\right) dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{3n+2}}{3n+2}\Big|_0^{.1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{(.1)^{3n+2}}{3n+2}$ 3 3

5

$\frac{.1^2}{2} - \frac{1}{1!}\frac{(.1)^5}{5} + \frac{(.1)^8}{2!(8)}$ 1

$< 10^{-6}$

Answer $\boxed{\frac{.01}{2} - \frac{.00001}{5}}$ 1

4. Use the fact that the following series is a telescoping series to calculate $\sum_{n=3}^{\infty} \frac{1}{n^2-1}$ exactly.

13

$\frac{1}{n^2-1} = \frac{A}{n-1} + \frac{B}{n+1} = \frac{1}{2}\left[\frac{1}{n-1} - \frac{1}{n+1}\right]$

4

$1 = A(n+1) + B(n-1)$

$n=1 \quad 1 = A(2) \Rightarrow A = \frac{1}{2}$

$1 = B(-2) \Rightarrow B = -\frac{1}{2}$

$\sum_{n=3}^{n-1} \frac{1}{2}\left[\frac{1}{n-1} - \frac{1}{n+1}\right]$

$\frac{1}{2}\left[\frac{1}{2} - \cancel{\frac{1}{4}}\right] + \frac{1}{2}\left[\frac{1}{3} - \cancel{\frac{1}{5}}\right] + \frac{1}{2}\left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right] + \frac{1}{2}\left[\cancel{\frac{1}{5}} - \frac{1}{7}\right] + \frac{1}{2}\left[\cancel{\frac{1}{6}} - \frac{1}{8}\right]$

3

4

$\lim S_n = \frac{1}{2}\left[\frac{1}{2} + \frac{1}{3}\right] = \frac{1}{2}\left[\frac{5}{6}\right] = \boxed{\frac{5}{12}}$

1 1

5. Calculate $\sum_{n=0}^{\infty} \frac{3^{2n}2^{n+2}}{5^{2n}}$ exactly.

$$2^2 \sum_{n=0}^{\infty} \left(\frac{18}{25}\right)^n = 2^2\left[\frac{\frac{18}{25}}{1-\frac{18}{25}}\right]$$

6. Use the integral test to determine the number of terms in the partial sum for $\sum_{n=1}^{\infty} \frac{1}{n^{11}}$ that will estimate the infinite series with an error less than 10^{-11}.

$$f(x) = \frac{1}{x^{11}}$$

$$R_n = \int_n^{+\infty} \frac{1}{x^{11}}\,dx = \lim_{b \to +\infty} -\frac{1}{10}x^{-10}\Big|_n^b = \frac{1}{10n^{10}}$$

$$\frac{1}{10n^{10}} < 10^{-11}$$

$$\frac{1}{n^{10}} < 10^{-10}$$

$$\frac{1}{n} < \frac{1}{10}$$

$$10 < n$$

$$n = 11$$

$$\therefore \boxed{\sum_{n=1}^{11} \frac{1}{n^{11}}}$$

MAT 162 Test 4 Name Key

Fall 2014 version 2

For full credit, show all work.

1. Tell why each series is conditionally convergent, absolutely convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(n+1)(n^2-1)}{100n^3-2n+1}$ diverges since $\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \frac{(n+1)(n^2-1)}{100n^3-2n+1} = \frac{1}{100} \neq 0$

12

6 answer

6 reason

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{7n}$ converges by Alt. Series Test

But $\sum_{n=1}^{\infty} \left|(-1)^n \frac{1}{7n}\right| = \frac{1}{7}\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (Harmonic Series)

12 $\therefore$ converges conditionally

(c) $\sum_{n=1}^{\infty} \frac{(-4)^{n+2}}{3^{2n+1}}$ $\frac{4^2}{3} \sum_{n=1}^{\infty} \left|\frac{(-4)^n}{9^n}\right|$ geometric series $|r| = \frac{4}{9}$

12 $\therefore$ absolutely convergent.

2. Find the radius and interval of convergence for $f(x) = \sum_{n=1}^{\infty}(x-5)^{2n}(16+4/n)^{-n}$.

$$\lim_{n\to+\infty}|a_n|^{\frac{1}{n}} = \lim_{n\to+\infty}\frac{|x-5|^2}{16+\frac{4}{n}} = \frac{|x-5|^2}{16} < 1 \quad 6$$

13

$|x-5| < 4$

$5-4 < x < 5+4$

$1 < x < 9$

$R = 4$

5

$\boxed{(1,9)}$

$x=9 \quad \sum_{n=1}^{\infty}\frac{4^{2n}}{(16+\frac{4}{n})^n} = \sum_{n=1}^{\infty}\left(\frac{16}{16+\frac{4}{n}}\right)^n = \sum_{n=1}^{\infty}\frac{1}{(1+\frac{1}{4n})^n}$

$\lim_{n\to+\infty}\frac{1}{(1+\frac{1}{4n})^n} = \frac{1}{e^{\frac{1}{4}}} \neq 0 \therefore$ diverges

$x=1 \quad \lim_{n\to+\infty}(-1)^n\left(\frac{16^n}{16+\frac{4}{n}}\right)^n \neq 0$ diverges

3. Use a power series to estimate $\int_0^{0.1} x\sin(x^3)\,dx$ with an error less than 10^{-6}.

$$13 \quad \int_0^{.1} x\sum_{n=0}^{\infty}\frac{(-1)^n(x^3)^{2n+1}}{(2n+1)!}dx = \int_0^{.1}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{x^{6n+4}}{} dx \quad 3$$

5

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{(.1)^{6n+5}}{6n+5} = \frac{1}{2(14)} \cdot \frac{1}{1614}$$

$$\frac{(.1)^5}{5} - \frac{(.1)^{11}}{3(11)} + \cdots \quad \therefore 10^{-6}$$

$.000002$

Answer $\frac{(.1)^5}{5} =$ .000002

4. Use the fact that the following series is a telescoping series to calculate $\sum_{n=3}^{\infty}\frac{1}{n^2-1}$ exactly.

$$13 \quad \frac{1}{n^2-1} = \frac{1}{2}\left[\frac{1}{n-1} - \frac{1}{n+1}\right] \text{ by Partial Fraction } 4$$

$$\sum_{n=3}^{\infty}\frac{1}{2}\left[\frac{1}{n-1}-\frac{1}{n+1}\right] = \frac{1}{2}\left[\frac{1}{2}-\cancel{\frac{1}{4}}\right]+\frac{1}{2}\left[\frac{1}{3}-\cancel{\frac{1}{5}}\right]+\frac{1}{2}\left[\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right]+\frac{1}{2}\left[\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right]$$

$$+\frac{1}{2}\left[\cancel{\frac{1}{6}}-\frac{1}{8}\right]+\frac{1}{2}\left[\cancel{\frac{1}{7}}-\frac{1}{9}\right]+\cdots$$

3 4

$$= \frac{1}{2}\left[\frac{1}{2}+\frac{1}{3}\right] = \frac{1}{2}\left[\frac{5}{6}\right] = \frac{5}{12}$$

1

1

5. Calculate $\sum_{n=0}^{\infty} \frac{3^{2n}2^{n+2}}{7^{2n}}$ exactly.

$2^2 \sum_{n=0}^{\infty} \left(\frac{18}{49}\right)^n = 4\left[\frac{1}{1-\frac{18}{49}}\right] = 4\left[\frac{49}{49-18}\right] = \frac{4(49)}{31}$

$= \frac{196}{31}$

≈ 6.322580645

12

6. Use the integral test to determine the number of terms in the partial sum for $\sum_{n=1}^{\infty} \frac{1}{n^{11}}$ that will estimate the infinite series with an error less than 5^{-10}.

$f(x) = \frac{1}{x^{11}}$

13

$|R_n(x)| \leq \int_n^{+\infty} \frac{1}{x^{11}}\,dx = \lim_{b \to +\infty} \int_n^b \frac{1}{x^{11}}\,dx$

$= \lim_{b \to +\infty} -\frac{1}{10} x^{-10} \Big|_n^b$

$= \frac{1}{10} n^{-10} < 5^{-10}$

$\frac{1}{10} 5^{10} < n^{10}$

$\left(\frac{1}{10}\right)^{\frac{1}{10}} 5 < n$

$3.97 < n$

Actually $\left(\frac{1}{10}\right)^{\frac{1}{10}} 5 < 5 \leq n$

$5 = n$

could take $n = 4$

$1.000494165 \quad \sum_{n=1}^{4} \frac{1}{n^4}$

$\therefore \boxed{\sum_{n=1}^{5} \frac{1}{n^{11}}}$