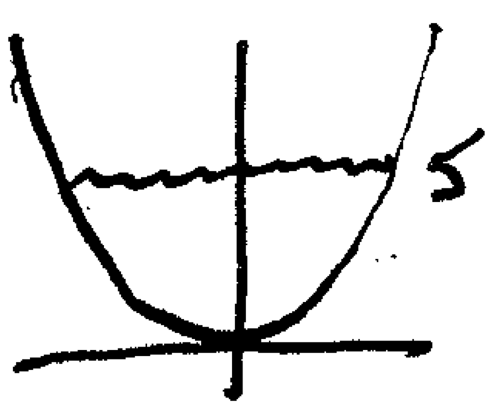
Fall, 2014 Name:______________________________

MAT 162001 Test 2 Version 1 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book. Each problem is worth 20 points. Problem 6 is 20 points extra credit.

1. A large tank is designed with ends in the shape of the region between the curves $y = (1/16)\,x^4$ and $y = 10$ measured in feet. Find the hydrostatic force on one end of the tank if it is filled to a depth of 5 ft. with gasoline. (Assume that the density of gasoline is 42.0 lb. per cubic foot.)



$y = \frac{1}{16}x^4$

$16y = x^4$ 2

$x = 2y^{\frac{1}{4}}$ 2

$\text{Area} = 2(2y^{\frac{1}{4}})\Delta y = 4y^{\frac{1}{4}}\Delta y$ 2

$\text{depth} = 5 - y$ 2

$\int_0^5 42(5-y)\,4y^{\frac{1}{4}}\,dy = (42)(4)\int_0^5 (5y^{\frac{1}{4}} - y^{5/4})\,dy$ 2

$= 42(4)\left(5\left(\tfrac{4}{5}\right)y^{5/4} - \tfrac{4}{9}y^{9/4}\right)\Big|_0^5$ 2

$= 168\left(4y^{5/4} - \tfrac{4}{9}y^{9/4}\right)\Big|_0^5$ 2

$= 168\left(4\cdot 5^{5/4} - \tfrac{4}{9}\cdot 5^{9/4}\right) = 2233.05418$

2. Find the length of the curve $y = 2x^{1.5} + 2$, $0 \le x \le 1$.

$f(x) = 2x^{1.5} + 2$

$f'(x) = 3x^{.5}$ (2)

$\sqrt{1 + |f'(x)|^2} = \sqrt{1+9x}$ (2)

$\int_0^1 \sqrt{1+|f'(x)|^2}\,dx = \int_0^1 \sqrt{1+9x}\,dx = (1+9x)^{3/2}\left(\tfrac{1}{9}\right)\tfrac{2}{3}\Big|_0^1$ (5) (2) (2) (5)

$= 10^{3/2}\left(\tfrac{2}{27}\right) - \tfrac{2}{27}$ (4)

$= 2.268353822$

3. Find the area of the surface obtained by rotating the curve $y = (1/3)x^3$, $1 \le x \le 2$, about the X-axis.

$f(x) = \frac{1}{3}x^3$

$f'(x) = x^2$

$$\int_1^2 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx = \int_1^2 2\pi \tfrac{1}{3}x^3\sqrt{1+x^4}\,dx$$

$w = 1+x^4$

$dw = 4x^3\,dx$

$\frac{1}{4}dw = x^3\,dx$

$$= \int_2^{17} \frac{2\pi}{3}\,\frac{1}{4}\,w^{\frac{1}{2}}\,dw$$

$$= \frac{\pi}{6}\,\frac{2}{3}\,w^{3/2}\Big|_2^{17}$$

$$= \frac{\pi}{9}\left(17^{3/2} - 2^{3/2}\right) = 23.479694$$

4. Suppose the average waiting time for a customer's call to be answered by a company representative (modeled by an exponentially decreasing probability density function) is 3 minutes. Find the median waiting time. Find the probability that is takes more than 7 minutes for the call to be answered.

4 $f(x) = \frac{1}{3}e^{-\frac{1}{3}x}$ if $x \ge 0$ 0 otherwise

Find M so that $\int_0^M \frac{1}{3}e^{-\frac{1}{3}x}\,dx = .5$ 2

$-e^{-\frac{1}{3}x}\Big|_0^M = .5$ 2

8

$1 - e^{-M/3} = .5$ 2

$.5 = e^{-M/3}$ 2

$\ln .5 = -\frac{M}{3}$ 2

$\boxed{-3\ln .5 = M} = 2.079441542$

$P(X>7) = 1 - P(X \le 7)$ 1

$= 1 - \int_0^7 \frac{1}{3}e^{-\frac{1}{3}x}\,dx$ 1

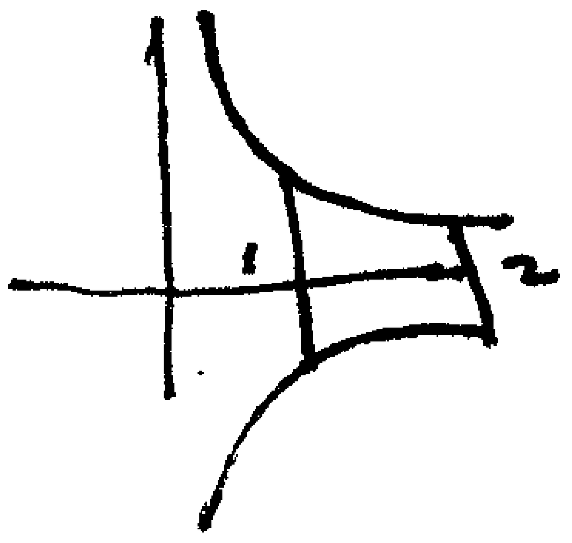
8

$= 1 - \left(-e^{\frac{1}{3}x}\Big|_0^7\right)$ 2

$= 1 - \left(1 - e^{-\frac{1}{3}7}\right)$ 2

$= \boxed{e^{-7/3}}$ $\approx$ ~~.0969~~ $.0969719679$ ~~.096971967~~

5. Find the centroid of the region bounded by the curves: $y = 1/x^2$, $y = -1/x^2$, $x = 1$ and $x = 2$.



4

$$M_{ass} = \int_1^2 \rho\left(\frac{1}{x^2} - -\frac{1}{x^2}\right) = 2\rho \int_1^2 \frac{1}{x^2}\,dx$$

$$= 2\rho\left(-\frac{1}{x}\Big|_1^2\right) = 2\rho\left[-\frac{1}{2}+1\right] = \rho$$

4 $$M_y = \int_1^2 \rho x\left(\frac{1}{x^2} - -\frac{1}{x^2}\right) = 2\rho \int_1^2 \frac{1}{x}\,dx$$

$$= 2\rho \ln x\Big|_1^2 = 2\rho \ln 2$$

4 $$M_x = \int_1^2 \frac{\rho}{2}\left(\left(\frac{1}{x^2}\right)^2 - \left(\frac{-1}{x^2}\right)^2\right)dx = 0$$

4 $$\bar{x} = \frac{M_y}{M} = 2\ln 2$$

4 $$\bar{y} = \frac{M_x}{M} = 0$$

6. Suppose $f(x) = ax^{-3} + bx^{-5}$ for $x \geq 3$ (for $x < 3$, $f(x) = 0$). Find a and b so that f(x) is a probability density function with a mean value of 4.

8

$$1 = \int_3^{+\infty} (ax^{-3} + bx^{-5})\,dx \overset{2}{=} \lim_{k\to+\infty} \int_3^k (ax^{-3} + bx^{-5})\,dx$$

✓

$$\overset{2}{=} \lim_{k\to+\infty} \left(\frac{a}{-2}x^{-2} + \frac{b}{-4}x^{-4}\right)\Big|_3^k$$

$$1 \overset{2}{=} \frac{a}{2}\frac{1}{9} + \frac{b}{4}\frac{1}{3^4}$$

so $4 \cdot 3^4 = 18a + b$

8

$$4 = \int_3^{+\infty} (ax^{-2} + bx^{-4})\,dx \overset{2}{=} \lim_{k\to+\infty} \int_3^k ax^{-2} + bx^{-4}\,dx$$

✓

$$\overset{2}{=} \lim_{k\to+\infty} \left(-ax^{-1} + \frac{b}{-3}x^{-3}\right)\Big|_3^c$$

$$\overset{2}{=} \frac{a}{3} + \frac{b}{3^4}$$

$3^4 \cdot 4 = 21a + b$

$0 = 3a \Rightarrow a = 0 \Rightarrow b = 3^4 \cdot 4$

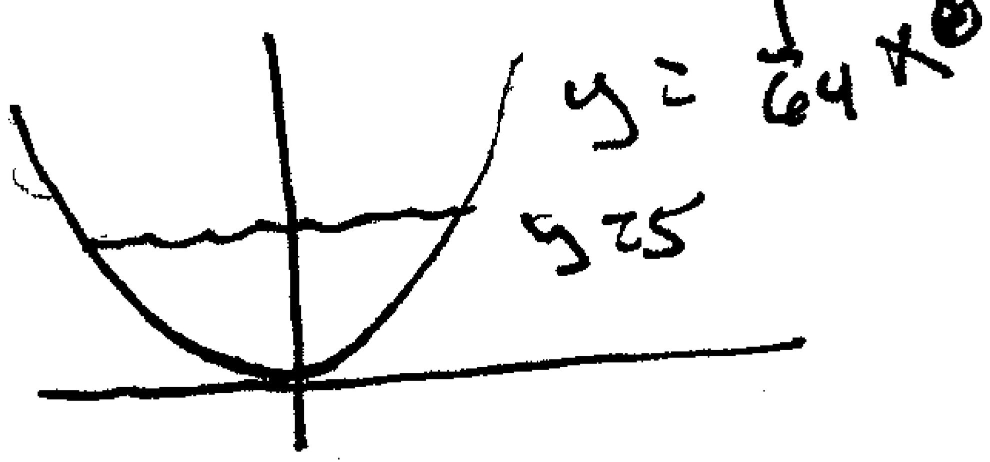
4

Fall, 2014 Name: Key

MAT 162001 Test 2 Version 2 Gurganus

Directions: Show all work for partial credit purposes. You may use a graphing calculator. Otherwise the test is closed book.

1. A large tank is designed with ends in the shape of the region between the curves y = (1/64)x^6 and y = 10 measured in feet. Find the hydrostatic force on one end of the tank if it is filled to depth of 5 ft. with gasoline. Assume that the density of gasoline is 42.0 lb. per cubic foot.)



$64y = x^6$

$2y^{\frac{1}{6}} = x$

Area $= 2(2y^{\frac{1}{6}})\Delta y$

depth $= 5-y$

$$\int_0^5 42(5-y)\,4y^{\frac{1}{6}}\,dy = 42(4)\int_0^5 (5y^{\frac{1}{6}} - y^{7/6})\,dy$$

$$= 42(4)\left(5\tfrac{6}{7}y^{7/6} - y^{13/6}\tfrac{6}{13}\right)\Big|_0^5$$

$$= 42(4)\left(5\left(\tfrac{6}{7}\right)5^{7/6} - \tfrac{6}{13}5^{13/6}\right)$$

$$= 42(4)\,5^{13/6}\left(\tfrac{6}{7} - \tfrac{6}{13}\right)$$

$$= 42(4)\,5^{13/6}\left[\tfrac{36}{7(13)}\right] = 2172.728192$$

2. Find the length of the curve y = $4x^{1.5}$, $1 \le x \le 3$.

$f(x) = 4x^{1.5}$

$f'(x) = 6x^{.5}$

$\sqrt{1+(f'(x))^2} = \sqrt{1+36x}$

$$\int_1^3 \sqrt{1+(f'(x))^2}\,dx = \int_1^3 \sqrt{1+36x}\,dx = \int_{37}^{109} \tfrac{1}{36} w^{\frac{1}{2}}\,dw$$

$$= \tfrac{1}{36}\tfrac{2}{3} w^{3/2}\Big|_{37}^{109}$$

$$= \tfrac{1}{(18)(3)}\left[109^{3/2} - 37^{3/2}\right]$$

16.9061332

$w = 1+36x$

$dw = 36\,dx$

$\frac{1}{36}dw = dx$

3. Find the area of the surface obtained by rotating the curve y = 3+5x , 1≤x≤2, about the X-axis.

$f(x) = 3+5x$

1 $f'(x) = 5$ 2

$$1 \int_1^2 2\pi f(x)\sqrt{1+f'(x)^2}\,dx = \int_1^2 2\pi(3+5x)\sqrt{26}\,dx$$

2 2

$$= \sqrt{26}\, 2\pi \left(3x + \frac{5}{2}x^2\right)\Big|_1^2$$

2

$$= 2\sqrt{26}\pi \left(3\cdot 2 + \frac{5}{2}(2)^2 - (3 + \frac{5}{2})\right)$$

$$= 2\sqrt{26}\pi \left[3 + 10 - \frac{5}{2}\right]$$

$$= 2\sqrt{26}\pi [10.5)$$

$$\approx \cancel{49.659}\ 336.399$$

4. Suppose the average waiting time for a customer's call to be answered by a company representative (modeled by an exponentially decreasing probability density function) is 2 minutes. Find the median waiting time. Find the probability that is takes more than 7 minutes for the call to be answered.

$f(x) = \frac{1}{2}e^{-\frac{1}{2}x} \quad x > 0$

8

$$.5 = \int_0^M \frac{1}{2}e^{-\frac{1}{2}x}\,dx = -e^{-\frac{1}{2}x}\Big|_0^M = 1 - e^{-\frac{M}{2}}$$

2 2 2

$$e^{-\frac{M}{2}} = .5$$

$$-\frac{M}{2} = \ln .5 \quad 2$$

$$M = -2\ln .5 \approx 1.386$$

8

$$1 - \int_0^7 \frac{1}{2}e^{-\frac{1}{2}x}\,dx = 1 + \left[e^{-\frac{1}{2}x}\Big|_0^7\right] \quad 2$$

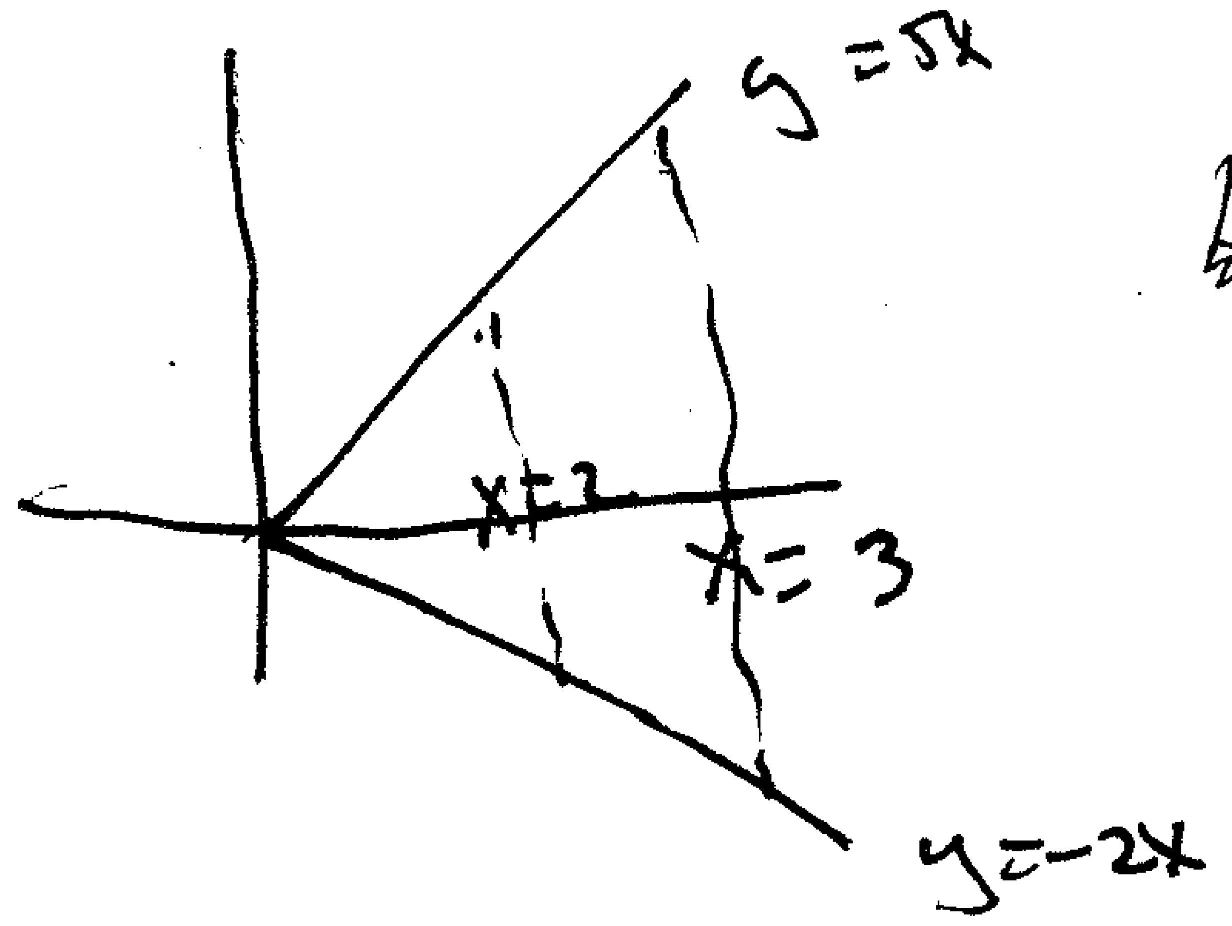
2

$$= 1 + e^{-\frac{7}{2}} - 1 \quad 2$$

$$= \boxed{e^{-\frac{7}{2}}} \quad 2$$

$$\approx .0301973$$

5. Find the centroid of the region bounded by the curves: y = 5x, y = -2x, x= 2 and x= 3.



4 $\text{Mass} = \int_2^3 \rho\,(5x - -2x)\,dx$

$= \rho\, 7\frac{x^2}{2}\Big|_2^3$

$= \frac{7}{2}\rho\,(9-4) = \frac{35\rho}{2}$

4 $M_y = \int_2^3 \rho\,(5x^2 + 2x^2)\,dx$

$\rho\, 7\,\frac{1}{3}x^3\Big|_2^3$

$\frac{7}{3}\rho\,(-2^3 + 3^3) = \frac{7}{3}\rho\,(+19) = \frac{133}{3}\rho$

$\bar{x} = \frac{M_y}{M} = \frac{\frac{7}{3}\rho(19)}{\frac{35\rho}{2}} = \frac{2\cdot 19}{15} = \boxed{\frac{38}{15}} \approx 2.5\bar{3}$

4 $M_x = \int_2^3 \rho\,\frac{1}{2}(25x^2 - 4x^2)\,dx$

$\rho\, 21\frac{x^3}{3}\Big|_2^3 = 7\rho\,\frac{(3^3 - 2^3)}{2} = \frac{133}{2}\rho$

$\bar{y} = \frac{M_x}{M} = \frac{7\rho(19)}{\frac{35}{2}\rho} = \frac{7(19)\cdot 2}{35} = \boxed{\frac{38}{5}}$

194/5

6. Suppose $f(x) = ax^{-3} + bx^{-5}$ for $x \geq 3$ (for $x < 3$, $f(x) = 0$). Find a and b so that f(x) is a probability density function with a mean value of 5.

$1 = \int_3^{+\infty} ax^{-3} + bx^{-5}\,dx \overset{2}{=} \lim_{b\to+\infty} \int_3^b ax^{-3} + bx^{-5}\,dx$ 2

$\overset{2}{=} \lim_{b\to+\infty} \left(\frac{a}{-2}x^{-2} + \frac{b}{-4}x^{-4}\right)\Big|_3^b$

$\overset{2}{=} \frac{a}{2}\frac{1}{9} + \frac{b}{4}\frac{1}{3^4}$

8

$4\cdot 3^4 = 18a + b$

$5 = \int_3^{+\infty} (ax^{-2} + bx^{-4})\,dx \overset{2}{=} \lim_{b\to+\infty} \int_3^b ax^{-2} + bx^{-4}\,dx$ 2

$\overset{2}{=} \lim_{b\to+\infty} -ax^{-1} + \frac{b}{-3}x^{-3}\Big|_3^b$

8

$\overset{2}{=} \frac{a}{3} + \frac{b}{3^4}$

4 $3^4\cdot 5 = 27a + b$

$3^4 = 9a \Rightarrow a = \frac{81}{9} \Rightarrow b = 4\cdot 81 - 18a = 4\cdot 81 - 18\left(\frac{81}{9}\right)$